



# DIGITAL ACCESS TO SCHOLARSHIP AT HARVARD

## Three Essays on Trading Behavior

The Harvard community has made this article openly available.  
[Please share](#) how this access benefits you. Your story matters.

<b>Citation</b>	Clark-Joseph, Adam Daniel. 2013. Three Essays on Trading Behavior. Doctoral dissertation, Harvard University.
<b>Accessed</b>	April 17, 2018 4:09:28 PM EDT
<b>Citable Link</b>	<a href="http://nrs.harvard.edu/urn-3:HUL.InstRepos:11156818">http://nrs.harvard.edu/urn-3:HUL.InstRepos:11156818</a>
<b>Terms of Use</b>	This article was downloaded from Harvard University's DASH repository, and is made available under the terms and conditions applicable to Other Posted Material, as set forth at <a href="http://nrs.harvard.edu/urn-3:HUL.InstRepos:dash.current.terms-of-use#LAA">http://nrs.harvard.edu/urn-3:HUL.InstRepos:dash.current.terms-of-use#LAA</a>

*(Article begins on next page)*

Three Essays on Trading Behavior

A dissertation presented

by

Adam Daniel Clark-Joseph

to

The Committee on Degrees in Economics

in partial fulfillment of the requirements

for the degree of

Doctor of Philosophy

in the subject of

Economics

Harvard University

Cambridge, Massachusetts

April 2013

© 2013 — Adam Daniel Clark-Joseph

All rights reserved.

Dissertation Advisor: Professor Andrei Shleifer

Adam Daniel Clark-Joseph

## Three Essays on Trading Behavior

### Abstract

This dissertation analyzes trading behavior in financial markets from multiple perspectives. In chapter 1, “Exploratory Trading,” I investigate the mechanisms underlying high-frequency traders’ capacity to profitably anticipate price movements. I develop a model of how a trader could gather valuable private information by using her own orders in an exploratory manner to learn about market conditions. The model’s predictions are borne out empirically, and I find that this “exploratory trading” model helps to resolve several central open questions about high-frequency trading. Chapters 2 and 3 focus on the trading behavior of individuals. Chapter 2, “Foundations of the Disposition Effect: Experimental Evidence,” (co-authored with Johanna Möllerström), presents and analyzes results from a laboratory experiment intended to examine if and how “regret aversion”—aversion to admitting mistakes—affects people’s trading decisions. Although the experimental results resolve little about regret aversion specifically, they reveal some novel and unexpected effects, most importantly that subjects radically changed their trading decisions when they were compelled to devote a minimal amount of extra attention. In chapter 3, “Price Targets,” I analyze how rational investors who privately observe information of indeterminate quality use prices to learn about whether or not their private information is valuable. I derive implications about trading behavior that not only help to explain a variety of empirical puzzles, but also generate several new testable predictions. Although these three essays differ considerably in methodology and focus, they all address the same basic issue of understanding the foundations of trading behavior.

# Contents

<b>Title Page</b>	<b>i</b>
<b>Abstract</b>	<b>iii</b>
<b>Acknowledgements</b>	<b>vi</b>
<b>1 Exploratory Trading</b>	<b>1</b>
1.1 Introduction . . . . .	1
1.2 Exploratory Trading: Theory . . . . .	5
1.3 High-Frequency Trading in the E-mini Market . . . . .	18
1.4 HFTs' Profits from Aggressive Orders . . . . .	21
1.5 Explicitly Isolating Exploratory Information . . . . .	30
1.6 Practical Significance of Exploratory Information . . . . .	39
1.7 Discussion . . . . .	44
1.8 Conclusion . . . . .	49
<b>2 Foundations of the Disposition Effect: Experimental Evidence</b>	<b>50</b>
2.1 Introduction . . . . .	50
2.2 Experimental Framework and Motivation . . . . .	55
2.3 Experimental Design: Details and Implementation . . . . .	65
2.4 Experimental Results and Analysis . . . . .	69
2.5 Discussion . . . . .	73

2.6	Conclusion . . . . .	76
<b>3</b>	<b>Price Targets</b>	<b>78</b>
3.1	Introduction . . . . .	78
3.2	A Rational Model With Price Targets . . . . .	81
3.3	Empirical Implications . . . . .	88
3.4	Foundations for Empirical Tests . . . . .	91
3.5	Discussion . . . . .	96
3.6	Conclusion . . . . .	100
<b>4</b>	<b>Appendices</b>	<b>101</b>
4.1	Chapter 1 Appendix . . . . .	101
4.2	Chapter 2 Appendix . . . . .	117
4.3	Chapter 3 Appendix . . . . .	139
	<b>Bibliography</b>	<b>147</b>

# Acknowledgements

I am grateful to Andrei Shleifer, John Campbell, Alp Simsek, Andrei Kirilenko and Jeremy Stein for giving me their invaluable advice, insight and support throughout the entire time that I have been fortunate enough to know each of them. I am also grateful to Brock Mendel for his detailed and insightful comments, feedback and discussion throughout my graduate career. I acknowledge financial support from the National Science Foundation Graduate Research Fellowship Program.

I thank Celso Brunetti, Richard Haynes, Todd Prono, Tugkan Tuzun, Brian Weller and Steve Yang for their assistance with the empirical component of “Exploratory Trading.” With respect to “Foundations of the Disposition Effect: Experimental Evidence,” I thank David Laibson and Al Roth for their helpful feedback, and David Smalling for his extensive assistance in developing the experimental software; I also acknowledge financial support from the Russell Sage Foundation and the Harvard Law School Program on Negotiations.

# Chapter 1

## Exploratory Trading<sup>1</sup>

### 1.1 Introduction

Over the past three decades, information technology has reshaped major financial exchanges worldwide. Physical trading venues have increasingly given way to electronic ones, and trading responsibilities that once fell on human agents have increasingly been delegated to computer algorithms. Automation now pervades financial markets; for example, Hendershott and Riordan (2009) and Hendershott *et al.* (2011) respectively document the dramatic levels of algorithmic trading on the Deutsche Boerse and the New York Stock Exchange. Much of the algorithmic activity in major markets emanates from so-called “high-frequency traders” (“HFTs”). Although it dominates modern financial exchanges, HFTs’ activity remains largely mysterious and opaque—it is the “dark matter” of the trading universe.

HFTs are distinguished not only by the large number of trades they generate (i.e., their literal high trading frequency), but also by the speed with which they can react to market events. HFTs achieve these remarkable reaction times, typically measured in milliseconds, by using co-location services, individual data feeds, and high-speed computer algorithms. Two further hallmarks of HFTs are their extremely short time-frames for maintaining positions, and their propensity for “ending the trading day in as close to a flat position as possible (that

---

<sup>1</sup>The views expressed in this chapter are my own and do not constitute an official position of the Commodity Futures Trading Commission, its Commissioners, or staff.



is, not carrying significant, unhedged positions over-night [when markets are closed]).”<sup>2</sup>

Empirical study of high-frequency trading has proven challenging, but not impossible. For example, Brogaard *et al.* (2012) obtain and analyze a NASDAQ dataset that flags messages from an aggregated group of 26 HFT firms, and Hasbrouck and Saar (2011) conduct a complementary analysis by statistically reconstructing “strategic runs” of linked messages in NASDAQ order data. Both of these analyses suggest beneficial effects from HFTs’ activity, but inherent limitations of the underlying data restrict these studies’ scope to explain how and why such effects arise.

Understanding and explaining the impacts of high-frequency trading requires some understanding of what HFTs are actually doing, and of how their strategies work. Even in a market for a single asset, HFTs exhibit considerable heterogeneity, so aggregate HFT activity reveals little about what individual HFTs really do. Data suitable for the study of individual HFTs’ activity are difficult to obtain. Whereas publicly available 13-F forms reveal the behavior of institutional investors at a quarterly frequency, there is no comparable public data that can be used to track and analyze the behavior of individual traders at a second- or millisecond-frequency.

The only fully adequate data currently available for academic research on high-frequency trading come from regulatory records that the Chicago Mercantile Exchange provides to the U.S. Commodity Futures Trading Commission. Kirilenko *et al.* (2010) pioneered the use of transaction data from these records to investigate high-frequency trading in their analysis of the so-called “Flash Crash” of May 6, 2010 in the market for E-mini Standard & Poors 500 stock index futures contracts (henceforth, “E-mini”). This work introduced a scheme to classify trading accounts using simple measures of overall trading activity, intraday variation in net inventory position, and inter-day changes in net inventory position. Of the accounts with sufficiently small intra- and inter-day variation in net position, Kirilenko *et al.* classify those with the highest levels of trading activity as HFTs, and these accounts are archetypes of high-frequency traders.

---

<sup>2</sup>U.S. Securities and Exchange Commission Concept Release No. 34-61358, pages 45-46.

Kirilenko *et al.* find that HFTs participate in over one-third of the trading volume in the E-mini market, and subsequent research by Baron *et al.* (2012) documents the large and stable profits that HFTs in the E-mini market earn. This work provides empirical confirmation of HFTs’ importance, and it offers some crisp descriptions of HFTs’ activity. However, it does not attempt to explain why HFTs act as they do, or how HFTs earn profits. Indeed, no extant empirical research attempts such explanations. In this paper, I address a central aspect of this open problem. HFTs in the E-mini market earn roughly 40% of their profits from the transactions that they initiate—that is, from their so-called “aggressive” orders—and I examine the mechanism underlying HFTs’ capacity to earn these profits.

How do HFTs in the E-mini market make money from their aggressive orders? One possibility is that HFTs merely react to public information faster than everyone else; this premise underlies the models of Biais *et al.* (2010), Jarrow and Protter (2011), and Cespa and Foucault (2008). A second possibility is that HFTs simply front-run coming demand when they can predict future aggressive orders. However, I find neither of these hypotheses to be consistent with the data.

I identify the HFTs who profit from their aggressive orders, then I investigate how these HFTs manage to do so. I show that the HFTs who profit from their aggressive trading use small aggressive orders to obtain private information that helps to forecast the price-impact of predictable demand innovations. Demand innovations in the E-mini market are easy to predict, but the price-elasticity of supply is not, and price-impact is usually too small for indiscriminate front-running of predictable demand to be profitable.<sup>3</sup> However, the private information about price-impact generated by an HFT’s small aggressive orders enables that HFT to trade ahead of predictable demand at only those times when it is profitable to do so (i.e., when price-impact is large). To elucidate how this works, I develop a theoretical model of what I term “exploratory trading.”

Fundamentally, the model of exploratory trading rests on the notion that an HFT’s aggres-

---

<sup>3</sup>To be more precise, it is extremely easy to predict whether future aggressive orders will be buy orders or sell orders. The dynamic behavior of passive orders resting in the orderbook—analogueous to supply elasticity—is considerably harder to forecast.

sive orders generate valuable private information, specifically, information about the price-impact of the aggressive orders that follow. When an HFT places an exploratory order and observes a large price-impact, he learns that supply is temporarily inelastic. If the HFT knows that there is going to be more demand soon thereafter, he can place a larger order (even with a big price-impact) knowing that the price-impact from the coming demand will drive prices up further and ultimately enable him to sell at a premium that exceeds the price-impact of his unwinding order. When an HFT knows that supply is temporarily inelastic, he follows a routine demand-anticipation strategy. The purpose of exploratory trading is not to learn about future demand, but rather to identify the times at which trading in front of future demand will be profitable. Active learning in financial markets is a relatively old idea, dating back at least to Leach and Madhavan’s papers in 1992 and 1993, but exploratory trading is an active-learning mechanism that is new to the academic literature.<sup>4</sup> In section 1.2, I present a model to formalize the concept of exploratory trading, and I derive the model’s central testable predictions.

Using novel electronic message data at the Commodity Futures Trading Commission, I examine the profitability of individual HFTs’ aggressive orders. I find that eight of the 30 HFTs in my sample profit from their aggressive trading overall and significantly outperform non-HFTs. However, these same eight HFTs all lose money on their smallest aggressive orders. (For brevity, I refer to these eight HFTs as “A-HFTs,” and to the remaining 22 as “B-HFTs.”) Exploratory trading would produce just such a pattern of incurring small losses on exploratory orders then realizing large gains; these descriptive results both motivate further tests and suggest the A-HFTs’ small aggressive orders as natural candidates for potential exploratory orders.

To explicitly test the predictions of the exploratory trading model for the eight A-HFTs, I examine the extent to which information about the changes in the orderbook following small aggressive orders explains the profits that various traders earn on subsequent aggressive

---

<sup>4</sup>Prior to Leach and Madhavan, Easley and Kiefer (1989) had examined the idea of learning from endogenous data in an abstract setting. Subsequent research, such as Moscarini and Smith (2001), further examines the abstract issue of active learning.

orders. The exploratory trading model predicts that information about the changes following an A-HFT's small aggressive order will explain a significant additional component of the A-HFT's subsequent performance, but that this information will not explain any additional component of other traders' subsequent performance. Consistent with these predictions, I find that the orderbook changes immediately following A-HFTs' small aggressive orders provide significant additional explanatory power for the respective A-HFTs' performance on their larger aggressive orders, but not for other traders' performance.

The remainder of this essay is organized as follows: Section 1.2 presents a simple model of exploratory trading, along with the model's central predictions, and establishes the empirical agenda. Section 1.3 describes the data and precisely defines HFTs. Section 1.4 addresses the overall profitability of HFTs' aggressive orders and precisely characterizes the A-HFTs, then examines the A-HFTs' losses on small aggressive orders. Section 1.5 presents direct empirical tests of the exploratory trading model's key predictions, section 1.6 examines the practical significance of exploratory information, and section 1.7 discusses extensions and implications of the empirical results. Section 1.8 concludes.

## **1.2 Exploratory Trading: Theory**

The ultimate objective of this paper is to explain the mechanism underlying HFTs' capacity to profit from their aggressive orders in the E-mini market, and this section establishes the theoretical framework for my empirical investigation.

As noted in the introduction, demand innovations in the E-mini market are easy to predict from public market data, but the price-elasticity of supply is not. Although there are times when supply is unaccommodating and high future demand forecasts price changes that are large enough to profit from, such times are difficult or impossible to identify by merely observing public market data. In this type of setting, a trader can obtain additional information about supply conditions by placing an "exploratory" aggressive order and observing how prices and supply respond. The additional exploratory information enables the trader to determine whether supply is accommodating (and expected price-impact small) or unaccommodating

(and expected price-impact large), and this helps the trader to decide whether he can profit by trading ahead of an imminent demand innovation.

The basic model I examine is a simple representation of a market in which demand is easy to predict, but supply elasticity is not. I consider a two-period model with two possible states for supply conditions (accommodating or unaccommodating), and three possible demand innovations in the second period (positive, negative, or zero). The demand innovation is automatically revealed before it arrives in the second period, but the state of supply conditions is only revealed if a trader places an aggressive order in the first period.

In this context, I consider the problem facing a single trader, the “HFT.” In the first period, the HFT has the opportunity to place an aggressive order and thereby learn about supply conditions. In the second period, regardless of what happened in the first period, the HFT observes a signal about future demand, after which he again has an opportunity to place an aggressive order. The signal of future demand forecasts price innovations much more accurately when combined with information about supply conditions than it does when used on its own. If the HFT places an aggressive order in the first period, he effectively “buys” supply information that he can use in the second period to better decide whether he should place another aggressive order. Consequently, the HFT may find it optimal in the first period to place an order that he expects to be unprofitable, since the information that the order generates will be valuable in the second period.

The rest of this section is devoted to formally developing a model of exploratory trading and deriving the model’s testable predictions. In addition to the basic result about the value of exploratory information sketched above, I address the key issue of why an order generates more information for the trader who submitted it than it does for everyone else. The appendix, section 4.1.1, contains full mathematical details.

### **1.2.1 Baseline Model**

In an order-driven market, every regular transaction is initiated by one of the two executing transactors. The transactor who initiates is referred to as the “aggressor,” while the opposite

transactor is referred to as the “passor.” The passor’s order was resting in the orderbook, and the aggressor entered a new order that executed against the passor’s preexisting resting order. Assuming that prices are discrete, the lowest price of any resting sell order in the orderbook (“best ask”) always exceeds the highest price of any resting buy order in the book (“best bid”) by at least one increment (the minimal price increments are called “ticks”). A transaction initiated by the seller executes at the best bid, while a transaction initiated by the buyer executes at the best ask; the resulting variation in transaction prices between aggressive buys and aggressive sells is known as “bid-ask bounce.” Hereafter, except where otherwise noted, I will restrict attention to price changes distinct from bid-ask bounce. Empirically, the best ask for the most actively traded E-mini contract almost always exceeds the best bid by exactly one tick during regular trading hours, so movements of the best bid, best ask, and mid-point prices are essentially interchangeable.

If the best bid and best ask were held fixed, a trader who aggressively entered then aggressively exited a position would lose the bid-ask spread on each contract, whereas a trader who passively entered then passively exited a position would earn the bid-ask spread on each contract. Intuitively, aggressors pay for the privilege of trading precisely when they wish to do so, and passors are compensated for the costs of supplying this “immediacy,” cf. Grossman and Miller (1988). These costs include fixed operational costs and costs arising from adverse selection. Cf. Glosten and Milgrom (1985), Stoll (1989).

An aggressive order will execute against all passive orders at the best available price level before executing against any passive orders at the next price, so an aggressive order will only have a literal price-impact if it eats through all of the resting orders at the best price. In the E-mini market, it is rare for an aggressive order to have a literal price-impact, not only because there are enormous numbers of contracts at the best bid and best ask, but also because aggressive orders overwhelmingly take the form of limit orders priced at the opposite best (which cannot execute at the next price level).

### 1.2.1.1 Market Structure

Let time be discrete, consisting of two periods,  $t = 1, 2$ . This model should be interpreted as a single instance of the hundreds or thousands of similar scenarios that arise throughout the trading day.

Consider an order-driven market with discrete prices, and assume that both the orderbook and order-flow are observable. Conceptually, the flow of aggressive orders is analogous to demand, while the set of passive orders in the orderbook (“resting depth”) is analogous to supply.<sup>5</sup>

### 1.2.1.2 Passive Orders

There are two possible states for the behavior of passive orders: accommodating and unaccommodating. Let the variable  $\Lambda$  represent this state, which I call the “liquidity state.” The liquidity state is the same in both periods of the model. Denote the accommodating liquidity state by  $\Lambda = A$ , and the unaccommodating state by  $\Lambda = U$ . Assume that  $\Lambda = U$  with *ex-ante* probability  $u$ , and  $\Lambda = A$  with complementary *ex-ante* probability  $1 - u$ .

The liquidity state characterizes the behavior of resting depth in the orderbook after an aggressive order executes—a generalization of price-impact appropriate for an order-driven market. When an aggressive buy (sell) order executes, it mechanically depletes resting depth on the sell (buy) side of the orderbook. Following this mechanical depletion, traders may enter, modify, and/or cancel passive orders, so resting depth at the best ask (bid) can either replenish, stay the same, or deplete further. The aggressive order’s impact is offset to some extent—or even reversed—if resting depth replenishes, whereas the aggressive order’s impact is amplified if resting depth depletes further. In the accommodating state ( $\Lambda = A$ ) resting depth weakly replenishes, while in the unaccommodating state ( $\Lambda = U$ ) resting depth further depletes. Intuitively, aggressive orders have a small price-impact in the accommodating state, and a large price-impact in the unaccommodating state.

---

<sup>5</sup>More precisely, the flow of aggressive orders and the set of passive orders literally constitute demand and supply, respectively, in the market for “immediacy.”

Although the orderbook is always observable, static features of passive orders in the orderbook do not directly reveal the liquidity state. Because the liquidity state relates to the dynamic behavior of resting depth after an aggressive order executes, this state can only be observed through the changes in the orderbook that follow the execution of an aggressive order.

### 1.2.1.3 Aggressive Order-Flow

At the end of period 2, traders other than the HFT exogenously place aggressive orders. Let the variable  $\varphi \in \{-1, 0, +1\}$  describe this exogenous aggressive order-flow. The variable  $\varphi$  is just a coarse summary of the order-flow—It does not represent the actual number of contracts. Intuitively,  $\varphi = -1$  represents predictable selling pressure and  $\varphi = +1$  represents predictable buying pressure, while  $\varphi = 0$  represents an absence of predictable pressure in either direction.

Assume that  $\varphi = +1$  and  $\varphi = -1$  with equal probabilities  $\mathbb{P}\{\varphi = +1\} = \mathbb{P}\{\varphi = -1\} = v/2$ , and  $\varphi = 0$  with complementary probability  $1 - v$ . The value of  $\varphi$  does not depend on the liquidity state,  $\Lambda$ , nor does it depend on the HFT's actions.

The price-change at the end of period 2, which I denote by  $y$ , is jointly determined by the exogenous aggressive order-flow and the liquidity state. In the notation of the model,

$$y = \begin{cases} \varphi & \text{if } \Lambda = U \\ 0 & \text{if } \Lambda = A \end{cases} \quad (1.1)$$

In other words, if the liquidity state is unaccommodating ( $\Lambda = U$ ), aggressive order-flow affects the price, and  $y = \varphi$ . However, if the liquidity state is accommodating ( $\Lambda = A$ ), aggressive order-flow does not affect the price, and  $y = 0$  regardless of the value of  $\varphi$ .

### 1.2.1.4 The HFT

The HFT submits only aggressive orders, and these aggressive orders are limited in size to  $N$  contracts or fewer. Let  $q_t \in \{-N, \dots, -1, 0, 1, \dots, N\}$  denote the signed quantity of the aggressive order that the HFT places in period  $t$ , where a negative quantity represents a sale,



and a positive quantity represents a purchase.

Assume that the HFT pays constant trading costs of  $\alpha \in (0.5, 1)$  per contract. The lower bound of 0.5 on  $\alpha$  corresponds to half of the minimum possible bid-ask spread, while the upper bound of 1 merely excludes trivial cases of the model in which aggressive orders are always unprofitable. When  $\alpha > u$ , the HFT will never place an order in period 2 if he doesn't know the liquidity state, and I focus on this case to simplify the exposition; results are qualitatively unchanged for  $u \geq \alpha$  (see the appendix, section 4.1.1).

I assume that the HFT's aggressive orders have no literal price-impact. Intuitively, the HFT only trades contracts at the initial best bid/ask. For example, in period 2, if the HFT has learned that the liquidity state is unaccommodating and  $\varphi = +1$ , he will buy all of the contracts available at the best ask. This is one way to interpret the size limitation on the HFT's orders.

The HFT's profit from the aggressive order he places in period  $t$  is given by

$$\pi_t = yq_t - \alpha |q_t| \tag{1.2}$$

where  $y$  denotes the price-change at the end of period 2. Let

$$\pi_{total} := \pi_1 + \pi_2$$

denote the HFT's total combined profits from periods 1 and 2. Assume that the HFT is risk-neutral and seeks to maximize the expectation of his total profits,  $\pi_{total}$ .

### 1.2.1.5 Model Timeline

**Period 1** In period 1, the HFT has the opportunity to submit an aggressive order and then observe any subsequent change in resting depth. The HFT cannot observe the liquidity state directly, but he can infer the value of  $\Lambda$  from changes in resting depth if he places an aggressive order; the HFT can conclude that  $\Lambda = u$  if resting depth further depletes following his order, and  $\Lambda = A$  otherwise. If the HFT does not place an aggressive order in period 1,

he does not learn  $\Lambda$ .

**Period 2** At the start of period 2, the HFT observes the signal of future aggressive order-flow,  $\varphi$ . The HFT observes  $\varphi$  regardless of whether he placed an aggressive order in period 1. After the HFT observes  $\varphi$ , he once again has an opportunity to place an aggressive order. Finally, after the HFT has the chance to trade, aggressive order-flow characterized by  $\varphi$  arrives, and prices change as determined by  $\varphi$  and  $\Lambda$  in equation (1.1).

Conceptually, the HFT's automatic observation of  $\varphi$  corresponds to the notion that aggressive order-flow is easy to predict on the basis of public market data. The HFT can always condition his period-2 trading strategy on  $\varphi$ , but he can condition this strategy on  $\Lambda$  only if he placed an aggressive order in period 1.

### 1.2.2 Exploratory Information is Valuable

The baseline model of exploratory trading illustrates why exploratory information can be valuable, and it highlights the trade-off between the direct costs of placing an exploratory order and the informational gains from exploration.

#### 1.2.2.1 Solving the Baseline Model

**Period 2** If the HFT learned the liquidity state during period 1, his optimal aggressive order in period 2 will depend on the values of both  $\varphi$  and  $\Lambda$ . The HFT's optimal strategy when he knows  $\Lambda$  is to set  $q_2 = \varphi N$  if  $\Lambda = U$ , and to set  $q_2 = 0$  if  $\Lambda = A$ . Taking expectations with respect to  $\varphi$  and then  $\Lambda$ , we find

$$\begin{aligned}\mathbb{E}[\pi_2 | \Lambda \text{ known}] &= Nv(1-\alpha)^*u + 0^*(1-u) \\ &= Nvu(1-\alpha)\end{aligned}\tag{1.3}$$

If the HFT did not learn the liquidity state during period 1, his (constrained) optimal aggressive order in period 2 will still depend on the value of  $\varphi$ , but it will only depend on the *distribution* of  $\Lambda$ , rather than the actual value of  $\Lambda$ . The HFT's optimal strategy when he

does not know  $\Lambda$  is to set  $q_2 = \varphi N$  when  $u \geq \alpha$ , and to set  $q_2 = 0$  when  $\alpha > u$ . I assumed for simplicity that  $\alpha > u$ , so

$$\mathbb{E}[\pi_2 | \Lambda \text{ unknown}] = 0 \quad (1.4)$$

**Period 1** At the start of period 1, the HFT knows neither  $\varphi$  nor  $\Lambda$ , but he faces the same trading costs ( $\alpha$  per contract) as in period 2. Consequently, the HFT's expected direct trading profits from a period-1 aggressive order are negative, and given by

$$\mathbb{E}[\pi_1] = -\alpha |q_1|$$

Since there is no noise in this baseline model, and the HFT learns  $\Lambda$  perfectly from any aggressive order that he places in the first period, we can restrict attention to the cases of  $q_1 = 0$  and  $|q_1| = 1$ .

We obtain the following expression for the difference in the HFT's total expected profits if he sets  $|q_1| = 1$  instead of  $q_1 = 0$ :

$$\mathbb{E}[\pi_{total} | |q_1| = 1] - \mathbb{E}[\pi_{total} | q_1 = 0] = Nvu(1 - \alpha) - \alpha \quad (1.5)$$

The HFT engages in exploratory trading if he sets  $|q_1| = 1$ , and he does not engage in exploratory trading if he sets  $q_1 = 0$ , so equation (1.5) represents the expected net gain from exploration. Exploratory trading is optimal for the HFT when this expected net gain is positive.

### 1.2.2.2 Conditions for Exploratory Trading

The results in section 1.2.2.1 demonstrate the trade-off between direct trading costs and informational gains at the heart of exploratory trading. By placing a (costly) aggressive order in period 1, the HFT “buys” the perturbation needed to elicit a response in resting depth that reveals the liquidity state. Knowing the liquidity state enables the HFT, in period 2, to better determine whether placing an aggressive order will be profitable. Parameters of the

model determine the relative costs and payoffs of exploration.

Recall that when the exogenous aggressive order-flow is described by  $\varphi = 0$ , the HFT does not have any profitable period-2 trading opportunities in either liquidity state. The probability that  $\varphi \neq 0$ , given by the parameter  $v$ , represents the extent to which the exogenous aggressive order-flow is predictable. To characterize how various parameters affect the viability of exploratory trading, I consider the minimal value of  $v$  for which the HFT finds it optimal to engage in period-1 (i.e., exploratory) trading. Denoting this minimal value by  $\underline{v}$ , we have

$$\underline{v} = \left(\frac{\alpha}{u}\right) \frac{1}{(1-\alpha)N} \quad (1.6)$$

The closer is  $\underline{v}$  to 0, the more conducive are conditions to exploratory trading.

The implications of equation (1.6) are intuitive. First, higher trading costs ( $\alpha$ ) tend to discourage exploratory trading. Second, when the HFT can use exploratory information to guide larger orders, the gains from exploration are magnified, so larger values of  $N$  tend to promote exploratory trading. Finally exploratory trading becomes less viable when  $u$  is smaller. The HFT will take the same action in period 2 when he knows that  $\Lambda = A$  as when he doesn't know  $\Lambda$ , so when  $u$  is small, knowledge of the liquidity state is less valuable because it is less likely to change the HFT's period-2 actions.<sup>6</sup>

Given the dearth of exogenous variation in the real-world analogues of  $\alpha$ ,  $N$  and  $u$ , the comparative statics above do not readily translate into empirically testable predictions. However, the model generates a much more fundamental prediction that can be tested empirically: *if an agent is engaging in exploratory trading, then the market response following his exploratory orders should help to explain his performance on subsequent aggressive orders.* The market response after a trader's exploratory orders should help to forecast price movements, and the trader will tend to follow up by placing further aggressive orders in the appropriate direction when the expected price movement is sufficiently large. Note that because the follow-up or-

---

<sup>6</sup>When  $u > \alpha$ , the HFT will take the same action in period 2 when he knows that  $\Lambda = U$  as when he doesn't know  $\Lambda$ , so knowledge of the liquidity state is less likely to change the HFT's period-2 actions when  $u$  is large. In the case of  $u > \alpha$ , equation (1.6) becomes  $\underline{v} = \frac{1}{(1-u)N}$ , and exploratory trading indeed becomes less viable as  $u$  approaches 1.

ders will tend to be larger than the exploratory orders, the market response after an agent's exploratory orders should help to explain not only the performance, but also the *incidence* of his larger aggressive orders.

### 1.2.3 Private Gains from Exploratory Trading

The baseline model of exploratory trading presented above abstracted away from the details of the HFT's inference about  $\Lambda$ . This simplifying assumption does not qualitatively affect the central result about the value of exploratory information, but it obscures why the HFT learns more from placing an aggressive order himself than he does from merely observing an aggressive order placed by someone else.

Factors other than aggressive order arrivals can affect the behavior of resting depth. In particular, a trader may adjust her passive orders in response to new information. Just as a trader might place an aggressive buy order if he believes that prices are too low, so might another trader who shared this belief cancel some of her passive sell orders. As a result, changes in resting depth are typically correlated with aggressive order-flow, even when the aggressive orders do not actually cause those changes. However, changes in resting depth not caused by aggressive orders do not help to forecast the price impact of future aggressive order-flow. The HFT learns more from aggressive orders that he places himself than he learns from those placed by other traders because he can better infer causal effects from his own orders.

#### 1.2.3.1 Intuition

An analogy to street traffic illustrates the main intuition for why the HFT obtains additional information from an aggressive order that he himself places. Consider a stoplight that tends to turn green shortly before a car arrives at it. This could arise for two reasons. First, the stoplight could operate on a timer, and cars might tend to approach the stoplight just before it turns green, due (e.g.) to the timing pattern of other traffic signals in the area. Alternatively, the stoplight might operate on a sensor that *causes* it to typically turn green when a car

approaches.

A driver who knows why she arrived at the stoplight at a certain time has a greater capacity to distinguish between the two explanations than does a pedestrian standing at the stoplight. In particular, if a driver knows that the moment of her arrival at the stoplight was not determined by the timing pattern of nearby traffic signals (e.g., if she had been parked, and the stoplight was the first traffic signal that she encountered), she will learn considerably more from her observation of the stoplight than will the pedestrian. Both pedestrian and driver can update their beliefs, but the pedestrian only weights the new observation by the average probability that the driver's arrival did not depend on the timing pattern of nearby signals.

Much as the driver's private knowledge about why she approaches the stoplight at a certain moment enables her to learn more than the pedestrian, the HFT's private knowledge of why he places an aggressive order enables him to learn more from the subsequent market response than he could learn from the response to an aggressive order placed by someone else.

### 1.2.3.2 Formalizing the Intuition

To make the preceding intuition more rigorous, consider a variant of the baseline model from section 1.2.1 in which some trader other than the HFT places an aggressive order at the beginning of period 1. With probability  $\rho$ , this aggressive order is the result of an unobservable informational shock, and resting depth further depletes following the order, regardless of the liquidity state  $\Lambda$ . Otherwise (with probability  $1 - \rho$ ) resting depth further depletes after the order if and only if the liquidity state is unaccommodating. Aside from this new aggressive order, all other aspects of the baseline model remain unchanged.

If the HFT places an aggressive order in period 1, his expected total profits are the same as they were in the baseline model, i.e.,

$$\mathbb{E}[\pi_{total} | |q_1| = 1] = Nvu(1 - \alpha) - \alpha$$

However, the HFT's expected profits if he does not place an order in period 1 are higher

than in the baseline model, because the HFT now learns something from the depth changes following the other trader's aggressive order. If resting depth weakly replenishes after that order, the HFT learns with certainty that the liquidity state is accommodating (i.e.,  $\Lambda = A$ ), so the HFT will not submit an aggressive order in period 2, and his total profits will be zero. Alternatively, if resting depth further depletes following the other trader's aggressive order, we have

$$\mathbb{P}\{\Lambda = U | \text{resting depth further depletes}\} = \frac{u}{u + \rho(1 - u)}$$

The HFT's optimal strategy when he does not know  $\Lambda$  is to set  $q_2 = \varphi N$  when  $\frac{u}{u + \rho(1 - u)} \geq \alpha$ , and to set  $q_2 = 0$  otherwise. Taking expectations with respect to  $\Lambda$  and  $\varphi$ , we find that the HFT's *ex-ante* expected total profits in this case are given by

$$\mathbb{E}[\pi_{total} | AO \text{ by someone else}] = \max \left\{ Nv \left( \frac{u}{u + \rho(1 - u)} - \alpha \right), 0 \right\} \quad (1.7)$$

### 1.2.3.3 Analysis

The features of the baseline model discussed in section 1.2.2.2 are qualitatively unchanged in the modified version, but now the “privacy” parameter  $\rho$  also exerts an influence. In the limiting case where the depth change following an aggressive order placed by someone else is completely uninformative to the HFT (i.e.,  $\rho = 1$ ), equation (1.7) collapses down to equation (1.4) from the baseline model. At the opposite extreme, when the HFT learns the liquidity state perfectly from observing another trader's aggressive order (i.e.,  $\rho = 0$ ), the HFT's expected total profits are unambiguously lower if he places an aggressive order in period 1 himself.

When the HFT can learn more about the liquidity state through mere observation, as he can when  $\rho$  is smaller, he has less incentive to incur the direct costs of exploratory trading. Viewed differently, if the HFT *does* find it optimal to engage in exploratory trading, it must be the case that he obtains more useful information from the market response to his aggressive orders than he does from the market response to other traders' aggressive orders. By symme-

try, it must also be the case that each other trader obtains no more useful information from the market response to the HFT's aggressive orders than they do from the market response to another arbitrary trader's aggressive orders.

#### **1.2.4 Testable Predictions**

Before attempting any empirical evaluation of the exploratory trading model's predictions, two basic issues must be addressed. First, it must be determined which HFTs, if any, actually earn positive and abnormal profits from their aggressive trading. I address this matter in section 1.4.2, and I identify eight such HFTs, to whom I refer as "A-HFTs." Next, among the A-HFTs' aggressive orders, suitable candidates for putative exploratory orders must be identified in some manner. The results from section 1.2.2.1 suggest that small, unprofitable aggressive orders are prime candidates. In section 1.4.4, I find that all of the A-HFTs, indeed, tend to lose money on their smallest aggressive orders, consistent with the theory that these orders are placed for exploratory ends.

With these two preliminary matters resolved, I turn to direct empirical tests of the model's key predictions. As a benchmark, I consider the market response following the last small aggressive order placed by anyone, which is public information. The empirical implications discussed earlier in this section can then be condensed into two central predictions, namely that relative to the public-information benchmark, the market response following an A-HFT's small aggressive order:

PREDICT.1. EXPLAINS A SIGNIFICANT ADDITIONAL COMPONENT OF THAT A-HFT'S EARNINGS ON  
SUBSEQUENT AGGRESSIVE ORDERS, BUT

PREDICT.2. DOES NOT EXPLAIN ANY ADDITIONAL COMPONENT OF OTHER TRADERS' EARNINGS ON  
SUBSEQUENT AGGRESSIVE ORDERS

In section 1.4.3, I make rigorous the notion of "explaining earnings on subsequent aggressive orders," then in section 1.5, I introduce an explicit numeric measure of "market response" and formally test the predictions above.



## 1.3 High-Frequency Trading in the E-mini Market

The E-mini S&P 500 futures contract is a cash-settled instrument with a notional value equal to \$50.00 times the S&P 500 index. Prices are quoted in terms of the S&P 500 index, at minimum increments, “ticks”, of 0.25 index points, equivalent to \$12.50 per contract. E-mini contracts are created directly by buyers and sellers, so the quantity of outstanding contracts is potentially unlimited.

All E-mini contracts trade exclusively on the CME Globex electronic trading platform, in an order-driven market. Transaction prices/quantities and changes in aggregate depth at individual price levels in the orderbook are observable through a public market-data feed, but the E-mini market provides full anonymity, so the identities of the traders responsible for these events are not released. Limit orders in the E-mini market are matched according to strict price and time priority; a buy (sell) limit order at a given price executes ahead of all buy (sell) limit orders at lower (higher) prices, and buy (sell) limit orders at the same price execute in the sequence that they arrived. Certain modifications to a limit order, most notably size increases, reset the time-stamp by which time-priority is determined.

E-mini contracts with expiration dates in the five nearest months of the March quarterly cycle (March, June, September, December) are listed for trading, but activity typically concentrates in the contract with the nearest expiration. Aside from brief maintenance periods, the E-mini market is open 24 hours a day, though most activity occurs during “regular trading hours,” namely, weekdays between 8:30 a.m. and 3:15 p.m. CT.

### 1.3.1 Description of the Data

I examine account-labeled, millisecond-timestamped records at the Commodity Futures Trading Commission of the so-called “business messages” entered into the Globex system between September 17, 2010 and November 1, 2010 for all E-mini S&P 500 futures contracts. This message data captures not only transactions, but also events that do not directly result in a trade, such as the entry, cancellation, or modification of a resting limit order. Essentially, business messages include any action by a market participant that could potentially result in

or affect a transaction immediately, or at any point in the future.<sup>7</sup> I restrict attention to the December-expiring E-mini contract (ticker ESZ0). During my sample period, ESZ0 activity accounted for roughly 98% of the message volume across all E-mini contracts, and more than 99.9% of the trading volume.

The price of an ESZ0 contract during this period was around \$55,000 to \$60,000, and (one-sided) trading volume averaged 1,991,252 contracts or approximately \$115 billion per day. Message volume averaged approximately 5 million business messages per day.

### 1.3.2 Defining “High-Frequency Traders”

Kirilenko *et al.* identify as HFTs those traders who exhibit minimal accumulation of directional positions, high inventory turnover, and high levels of trading activity. I, too, use these three characteristics to define and identify HFTs. To quantify an account’s accumulation of directional positions, I consider the magnitude of changes in end-of-day net position as a percentage of the account’s daily trading volume. Similarly, I use an account’s maximal intraday change in net position, relative to daily volume, to measure inventory turnover. Finally, I use an account’s total trading volume as a measure of trading activity.

I select each account whose end-of-day net position changes by less than 6% of its daily volume, and whose maximal intraday net position changes are less than 20% of its daily volume. I rank the selected accounts by total trading volume, and classify the top 30 accounts as HFTs. The original classifications of Kirilenko *et al.* and Baron *et al.* guided the rough threshold choices for inter-day and intraday variation. Thereafter, since confidentiality protocols prohibit disclosing results for groups smaller than eight trading accounts, the precise cutoff values of 6%, 20%, and 30 accounts were chosen to ensure that all groups of interest would have at least eight members. My central results are not sensitive to values of these parameters.

---

<sup>7</sup>Excluded from these data are purely administrative messages, such as log-on and log-out messages. The good-’til-cancel orders in the orderbook at the start of September 2, and a small number of modification messages (around 2 – 4%) are also missing from these records. Because I restrict attention to aggressive orders, and I only look at *changes* in resting depth (rather than its actual level), my results are not sensitive to these omitted messages.

The set of HFTs corresponds closely to the set of accounts with the greatest trading volume in my sample, so the set of HFTs is largely invariant both to the exact characterizations of inter-day and intraday variation in net position relative to volume, and to the exact cutoff values for these quantities. Similarly, changing the 30-account cutoff to (e.g.) 15 accounts or 60 accounts does not substantially alter my results, because activity heavily concentrates among the largest HFTs. For example, the combined total trading volume of the 8 largest HFTs exceeds that of HFTs 9-30 by roughly three-quarters, and the combined aggressive volume of the 8 largest HFTs exceeds that of HFTs 9-30 by a factor of almost 2.5.

### **1.3.3 HFTs' Prominence and Profitability**

Although HFTs constitute less than 0.1% of the 41,778 accounts that traded the ESZ0 contract between September 17, 2010 and November 1, 2010, they participate in 46.7% of the total trading volume during this period. In addition to trading volume, HFTs are responsible for a large fraction of message volume. During the sample period, HFTs account for 31.9% of all order entry, order modification and order cancellation messages. The HFTs also appear to earn large and stable profits. Gross of trading fees, the 30 HFTs earned a combined average of \$1.51 million per trading day during the sample period. Individual HFTs' annualized Sharpe ratios are in the neighborhood of 10 to 11.

The Chicago Mercantile Exchange reduces E-mini trading fees on a tiered basis for traders whose average monthly volume exceeds various thresholds. Trading and clearing fees were either \$0.095 per contract or \$0.12 per contract for the 20 largest HFTs, and were at most \$0.16 per contract for the remaining HFTs. Initial and maintenance margins were both \$4,500 for all of the HFTs.

Hereafter, unless otherwise noted, I restrict attention to activity that occurred during regular trading hours. HFTs' aggressive trading occurs almost exclusively during regular trading hours (approximately 95.6%, by volume), and market conditions during these times differ substantially from those during the complementary off-hours.

## 1.4 HFTs' Profits from Aggressive Orders

Aggressive trading is a tremendously important component of HFTs' activity. In aggregate, approximately 48.5% of HFTs' volume is aggressive, and this figure rises to 54.2% among the 12 largest HFTs. Furthermore, many HFTs consistently profit from their aggressive trading. Since the bid-ask spread in the E-mini market rarely exceeds the minimum imposed upon it by the granularity of prices, there is little mystery about how a trader's passive trades could consistently earn money.<sup>8</sup> By contrast, explaining how a trader who uses only market data could consistently profit on aggressive trades is somewhat difficult.

### 1.4.1 Measuring Aggressive Order Profitability

Because all E-mini contracts of a given expiration date are identical, it is neither meaningful nor possible to distinguish among the individual contracts in a trader's inventory, so there is generally no way to determine the exact prices at which a trader bought and sold a particular contract. As a result, it is typically impossible to measure directly the profits that a trader earns on an individual aggressive order. However, the cumulative price change following an aggressive order, normalized by the order's direction (+1 for a buy, or  $-1$  for a sell), can be used to construct a meaningful proxy for the order's profitability. Intuitively, the average expected profit from an aggressive order equals the expected favorable price movement, minus trading/clearing fees and half the bid-ask spread. See the appendix, section 4.1.2 for rigorous justification.

Estimating the cumulative favorable price movement after an aggressive order is straightforward. Consider a trader who can forecast price movements up to  $j$  time periods in the future, but no further. If the trader places an aggressive order in period  $t$ , any price changes that she could have anticipated at the time she placed the order will have occurred by period  $t + j + 1$ . Provided that price is a martingale with respect to its natural filtration, the expected

---

<sup>8</sup>Explaining the profitability of individual passive trades does not resolve the question of how various HFTs manage to participate in so many passive trades. In equilibrium, we would expect new entrants to reduce the average passive volume of an individual trader until her total profits from passive trades equaled her fixed costs.

change in price from period  $t + j + 1$  onward is zero, both from the period- $t$  perspective of the trader and from an unconditional perspective. Thus the change in price between period  $t$  and any period after  $t + j$ , normalized by the direction of the trader's order (+1 for a buy, or -1 for a sell), will provide an unbiased estimate of the favorable price movement following the trader's order.

The remarks above imply that we can derive a proxy for the profitability of an HFT's aggressive order using the (direction-normalized) accumulated price-changes following that aggressive order out to some time past the HFT's maximum forecasting horizon. If we choose too short an accumulation window, the resulting estimates of the long-run direction-normalized average price changes following the HFT's aggressive orders will be biased downward. As a result, we can empirically determine an adequate accumulation period by calculating cumulative direction-normalized price changes over longer and longer windows until their mean ceases to significantly increase. Using too long an accumulation period introduces extra noise, but it will not bias the estimates. I find that an accumulation period, measured in event-time, of 30 aggressive order arrivals is sufficient to obtain unbiased estimates; for all of the empirical work in this paper, I use an accumulation period of 50 aggressive order arrivals to allow a wide margin for error. See the appendix, section 4.1.2, for further details.

As noted earlier, the bid-ask spread for the E-mini is almost constantly \$12.50 (one tick) during regular trading hours, and the HFTs in my sample face trading/clearing fees of \$0.095 to \$0.16 per contract, so the average favorable price movement necessary for an HFT's aggressive order to be profitable is between \$6.345 and \$6.41 per contract. Since trading/clearing fees vary across traders, I report aggressive order performance in terms of favorable price movement, that is, earnings gross of fees and the half-spread.

#### **1.4.2 HFTs' Overall Profits from Aggressive Orders**

To measure the overall mean profitability of a given account's aggressive trading, I compute the average cumulative price change following each aggressive order placed by that account, weighted by executed quantity and normalized by the direction of the aggressive order. As a

group, the 30 HFTs in my sample achieve size-weighted average aggressive order performance of \$7.01 per contract. On an individual basis, nine HFT accounts exceed the relevant \$6.25 + *fees* profitability hurdle, and each of these nine accounts exceeds this hurdle by a margin that is statistically significant at the 0.05 level. One of these nine accounts is linked with another HFT account, and their combined average performance also significantly exceeds the profitability hurdle.

Overall, the HFTs vastly outperform non-HFTs, who earn a gross average of \$3.19 per aggressively-traded contract. However, these overall averages potentially confound effects of very coarse differences in the times at which traders place aggressive orders with effects of the finer differences more directly related to strategic choices. For example, if all aggressive orders were more profitable between 1 p.m. and 2 p.m. than at other times, and HFTs only placed aggressive orders during this window, the HFTs' outperformance would not depend on anything characteristically high-frequency.

To control for potential low-frequency confounds, I divide each trading day in my sample into 90-second segments and regress the profitability of non-HFTs' aggressive orders during each segment on both a constant and the executed quantities of the aggressive orders. Using these local coefficients, I compute the profitability of each aggressive order by an HFT in excess of the expected profitability of a non-HFT aggressive order of the same size during the relevant 90-second segment. With these additional controls, only 27 HFT accounts continue to exhibit significant outperformance of non-HFTs, and only eight of the 27 accounts are among those whose absolute performance exceeded the profitability hurdle.

#### **1.4.2.1 A-HFTs and B-HFTs**

For expositional ease, I will refer to the eight HFT accounts that make money on their aggressive trades *and* outperform the time-varying non-HFT benchmark as "A-HFTs," and to the complementary set of HFTs as "B-HFTs." The eight A-HFTs have a combined average daily trading volume of 982,988 contracts, and on average, 59.2% of this volume is aggressive. The 22 B-HFTs have a combined average daily trading volume of 828,924 contracts, of which

35.9% is aggressive. Gross of fees, the A-HFTs earn a combined average of \$793,342 per day, or an individual average of \$99,168 per day, while the B-HFTs earn a combined average of \$715,167 per day, or an individual average of \$32,508 per day.<sup>9</sup> The highest profitability hurdle among the A-HFTs is \$6.37 per aggressively traded contract.

### **1.4.3 Relative Aggressive Order Profitability: HFT vs. Econometrician**

To gain some insight into the factors that affect aggressive trading profits, I examine the extent to which econometric price forecasts explain the realized performance of aggressive orders placed by A-HFTs, B-HFTs, and non-HFTs. The methodology that I develop in this subsection also provides the starting point for my direct tests of the exploratory trading model's predictions in section 1.5.

#### **1.4.3.1 Variables that Forecast Price Movements**

Bid-ask bounce notwithstanding, the price at which aggressive orders execute changes rather infrequently in the E-mini market. On average, only about 1 – 3% of aggressive buy (sell) orders execute at a final price different from the last price at which the previous aggressive buy (sell) order executed, and the price changes that do occur are almost completely unpredictable on the basis of past price changes. However, several other variables forecast price innovations surprisingly well.

In contrast to price innovations, the direction of aggressive order flow in the E-mini market is extremely persistent. On average, the probability that the next aggressive order will be a buy (sell) given that the previous aggressive order was a buy (sell) is around 75%. In addition to forecasting the direction of future aggressive order flow, the direction of past aggressive order flow also forecasts future price innovations to statistically and economically significant extent, and forecasts based on past aggressive order signs alone are modestly improved by information about the (signed) quantities of past aggressive orders. Price forecasts can be further improved using simple measures of recent changes in the orderbook.

---

<sup>9</sup>All of the preceding descriptive statistics include the small amount of trading activity that occurred outside regular trading hours.

### 1.4.3.2 Econometric Benchmark

For each trading day in my sample, I regress the cumulative price-change (in dollars) between the aggressive orders  $k$  and  $k + 50$ , denoted  $y_k$ , on lagged market variables suggested by the remarks above. Specifically, I regress  $y_k$  on the changes in resting depth between aggressive orders  $k - 1$  and  $k$  at each of the six price levels within two ticks of the best bid or best ask, the signs of aggressive orders  $k - 1$  through  $k - 4$ , and the signed executed quantities of aggressive orders  $k - 1$  through  $k - 4$ . For symmetry, I adopt the convention that sell depth is negative, and buy depth is positive, so that an increase in buy depth has the same sign as a decrease in sell depth. Denoting the row vector of the 14 regressors by  $z_{k-1}$ , and a column vector of 14 coefficients by  $\Gamma$ , I estimate the equation

$$y_k = z_{k-1}\Gamma + \epsilon_k \quad (1.8)$$

The chapter appendix presents coefficient estimates and direct discussion of the regression results.

To compute the excess performance of aggressive order  $k$ , denoted  $\xi_k$ , I normalize the  $k$ th regression residual by  $sign_k$ , the sign of the  $k$ th aggressive order:

$$\xi_k = sign_k \left( y_k - z_{k-1}\hat{\Gamma} \right)$$

As discussed in section 1.4.1, normalizing the cumulative price-change  $y_k$  by the sign of the  $k$ th aggressive order yields a measure of the  $k$ th aggressive order's profitability. Likewise, the quantity  $\xi_k$  provides a measure of  $k$ th aggressive order's profitability in excess of that expected on the basis of the benchmark econometric specification. I compute the vectors of direction-normalized residuals separately for each of the 32 trading days in my sample, then combine all of them into a single vector for the entire sample period.



### 1.4.3.3 Explained Performance

The price movements predicted by (1.8) explain a substantial component of the performance of aggressive orders placed by A-HFTs, B-HFTs, and non-HFTs alike.<sup>10</sup> Looking ahead, this explanatory power validates the use of specification (1.8) as a basis for the more sophisticated analyses in section 1.5. Figure 1.1 and Table 1.1 summarize the overall size-weighted average performance of aggressive orders placed by various trader groups, both in absolute terms, and in excess of the econometric benchmark. Confidence intervals are computed via bootstrap.

Table 1.1: Aggressive Order Performance vs. Econometric Benchmark

	Mean Absolute	Mean Absolute 99% CI	Mean Excess	Mean Excess 99% CI	Explained Performance
A-HFTs	7.65	(7.54, 7.73)	3.22	(3.09, 3.36)	57.8%
B-HFTs	5.67	(5.57, 5.77)	2.04	(1.90, 2.17)	64.0%
Non-HFTs	3.19	(3.12, 3.26)	0.22	(0.14, 0.29)	93.06%

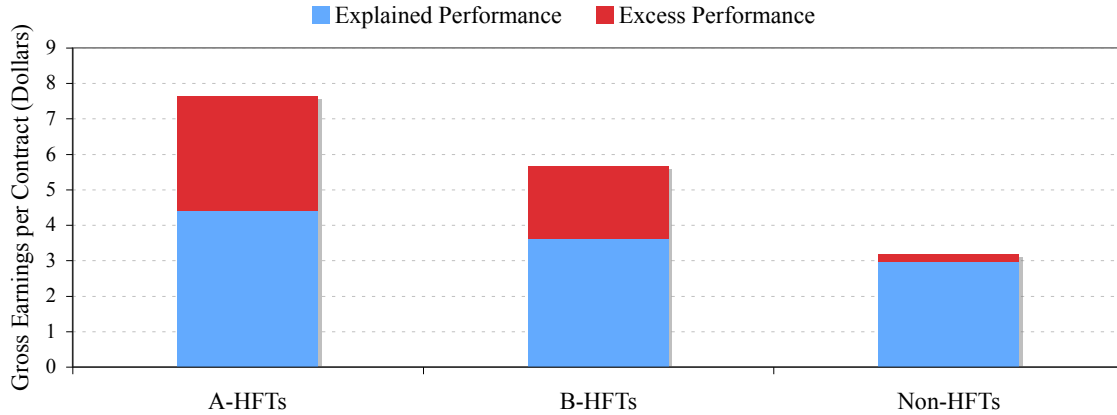


Figure 1.1: Aggressive Order Performance Relative to Econometric Benchmark

<sup>10</sup>Although the variables in  $z_{k-1}$  are all observable before the  $k$ th aggressive order arrives, the fitted value  $z_{k-1}\hat{\Gamma}$  is not literally a forecast of  $y_k$  in the strictest sense, as  $\hat{\Gamma}$  is estimated from data for the entire day. However, the coefficient estimates are extremely stable throughout the sample period, so thinking of  $z_{k-1}\hat{\Gamma}$  as a forecast of  $y_k$  is innocuous in the present setting. See section 1.6.1.

The exploratory trading model developed earlier assumed that A-HFTs ultimately traded ahead of easily predictable demand innovations (when liquidity conditions were suitably unaccommodating), and the explanatory power of equation (1.8) for the A-HFTs' performance substantiates this assumption. At the same time, although the econometric controls explain over half of the A-HFTs' performance on their aggressive orders, the remaining unexplained component of performance is massive. The A-HFTs' average excess performance is over 50% greater than that of the B-HFTs, and over 10 times greater than that of the non-HFTs. Price forecasts more sophisticated than those from (1.8) may better explain A-HFTs' performance; I return to this matter in section 1.5.

#### 1.4.4 A-HFTs' Losses on Small Aggressive Orders

A-HFTs' aggressive orders tend to become more profitable as order size increases.<sup>11</sup> In fact, despite earning money from their aggressive orders on average, the A-HFTs all tend to *lose* money on the smallest aggressive orders that they place. Note also that I refer here to the size of the aggressive orders A-HFTs submit, not the quantity that executes, so the small orders were intentionally chosen to be small, and the large orders intentionally chosen to be large.

The baseline model of exploratory trading in section 1.2 produces exactly the sort of losses on small aggressive orders and profits on large aggressive orders that the A-HFTs exhibit. The A-HFTs' differing performance on small and large aggressive orders is consistent with the pattern that we would expect to see if the small orders were generating valuable information that enabled the A-HFTs to earn greater profits from their large orders.

To make precise both the meaning of “small” aggressive orders, and A-HFTs' losses on them, I specify cutoffs for order size and compute the average performance of A-HFTs' aggressive orders below and above those size cutoffs. Figure 1.2 and Table 1.2 display bootstrap confidence intervals for the executed-quantity-weighted average performance of A-HFTs' aggressive orders weakly below and strictly above various order-size cutoffs.

---

<sup>11</sup>This effect appears whether price-changes are measured between the respective last prices at which successive aggressive orders execute (correcting for bid-ask bounce), or between the respective first prices at which they execute, so the positive relationship between executed quantity and subsequent favorable price movements is not simply an artifact of large orders that eat through one or more levels of the orderbook.

Table 1.2: Performance of A-HFTs' Aggressive Orders (Dollars per Contract)

Cutoff	Below Cutoff 95% CI	Above Cutoff 95% CI	AOs Below Cutoff % of All AOs	AOs Below Cutoff % of Aggr. Volume
1	(3.78, 3.89)	(7.59, 7.74)	24.31%	0.40%
5	(4.17, 4.29)	(7.62, 7.78)	43.74%	1.44%
10	(3.42, 3.55)	(7.71, 7.85)	54.64%	3.09%
15	(3.79, 3.92)	(7.71, 7.86)	56.75%	3.54%
20	(4.08, 4.20)	(7.75, 7.90)	60.82%	4.80%

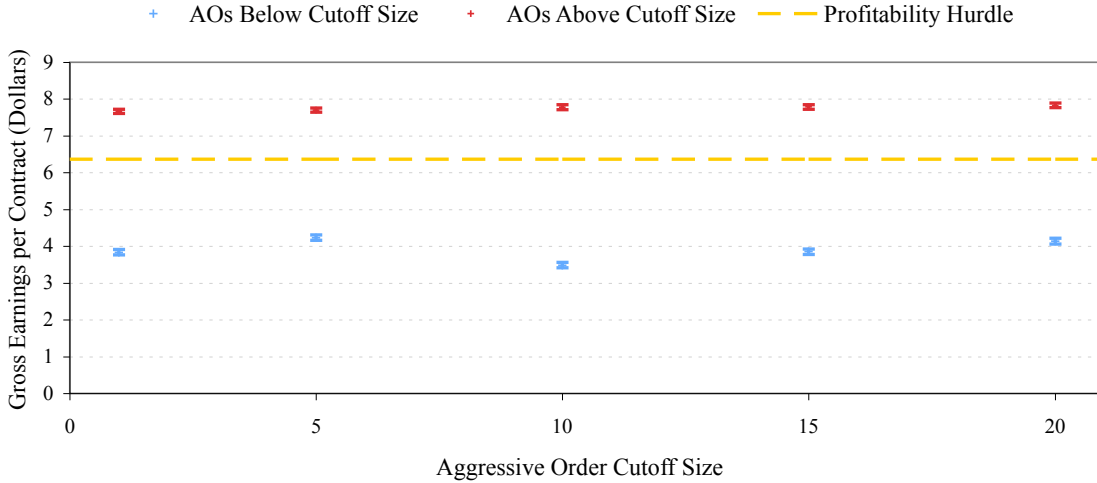


Figure 1.2: A-HFT Performance on Small and Larger Aggressive Orders (95% Conf. Intervals)

As shown in Table 1.2, small aggressive orders represent a substantial fraction of the aggressive orders that A-HFTs place, but these small orders make up very little of the A-HFTs' total aggressive volume. Nevertheless, A-HFTs' losses on these small orders are non-negligible. On average, each A-HFT loses roughly \$7,150 per trading day (\$1.8 million, annualized) on aggressive orders of size 20 or less; this loss represents approximately 7.2% of an average A-HFT's daily profits.

Although HFTs may tolerate only limited levels of inventory, inventory-management does not adequately explain the A-HFTs’ losses on small aggressive orders. We can control for A-HFTs’ respective net positions at the times they submit aggressive orders, and restrict attention to only those aggressive orders that move an A-HFT *away* from a zero net position. Such “non-rebalancing” orders account for over half of the small aggressive orders that A-HFTs place, and they cannot possibly be motivated by inventory management. Nevertheless, A-HFTs still lose money on the smallest of these orders, yet make money on the larger ones. Table 1.3 summarizes the performance of aggressive orders that move the submitting account *away* from a neutral inventory position. A net position of zero is the most likely “neutral” inventory target, but I allow for the possibility that a given HFT has an arbitrary constant target inventory position, and I restrict attention to aggressive orders that move the HFT away from that target inventory level.

Table 1.3: Performance of A-HFTs’ Non-Rebalancing Aggressive Orders

Cutoff	Below Cutoff 95% CI	Above Cutoff 95% CI	AOs Below Cutoff % of All AOs	AOs Below Cutoff % of Aggr. Volume
1	(3.24, 3.42)	(6.90, 7.12)	15.00%	0.25%
5	(3.64, 3.79)	(6.95, 7.17)	27.25%	0.89%
10	(2.48, 2.63)	(7.13, 7.35)	34.06%	1.94%
15	(2.79, 2.96)	(7.13, 7.35)	35.01%	2.14%
20	(3.01, 3.19)	(7.21, 7.44)	37.16%	2.81%

The A-HFTs’ qualitative pattern of losses on small aggressive orders and more-than-offsetting gains on larger aggressive orders suggests that the small orders are reasonable candidates for exploratory orders. This finding provides a foundation for direct tests of the exploratory trading model’s sharper empirical predictions.

## 1.5 Explicitly Isolating Exploratory Information

If the exploratory trading model is correct, and if the A-HFTs' small aggressive orders are indeed exploratory in nature, the two key model predictions presented in section 1.2.4 must hold. For convenience, I summarize these predictions below.

Relative to a benchmark that incorporates the public information about the market response following small aggressive orders placed by anyone, the market response following small aggressive orders placed by an A-HFT:

PREDICT.1. EXPLAINS A SIGNIFICANT ADDITIONAL COMPONENT OF THAT A-HFT'S EARNINGS ON  
SUBSEQUENT AGGRESSIVE ORDERS, BUT

PREDICT.2. DOES NOT EXPLAIN ANY ADDITIONAL COMPONENT OF OTHER TRADERS' EARNINGS ON  
SUBSEQUENT AGGRESSIVE ORDERS

In this section, I consider a simple numeric characterization of the market response following an aggressive order, and I directly test whether the above predictions of the exploratory trading model hold. I estimate results for the A-HFTs individually, but for compliance with confidentiality protocols, I present cross-sectional averages of these estimates. Empirically, these average results are representative of the results for individual A-HFTs.<sup>12</sup>

### 1.5.1 Empirical Strategy: Overview

Though the implementation is slightly involved, my basic empirical strategy is straightforward. First, I augment the benchmark regression from section 1.4.3 using

1. Market response information from the last small aggressive order placed by anyone, and
2. Both market response information from the last small aggressive order placed by anyone,  
AND market response information from the last small aggressive order *placed by a specified A-HFT*

---

<sup>12</sup>Throughout the E-mini market, there exist assorted linkages between various trading accounts (as, for example, in the simple case where single firm trades with multiple accounts), so the trading-account divisions do not necessarily deliver appropriate atomic A-HFT units. Though the specifics are confidential, the appropriate partition of the A-HFTs is entirely obvious. For brevity, I use "individual A-HFT" as shorthand to "individual atomic A-HFT unit," as applicable.

As I discuss in more detail in the next subsection, the market-response variable that I consider essentially amounts to a measure of the change in orderbook depth that follows an aggressive order.

After estimating both of the specifications above, I find the additional component of performance on larger aggressive orders explained by (2) relative to (1). The market response following an arbitrary small aggressive order is publicly observable. However, because the E-mini market operates anonymously, the distinction between a small aggressive order placed by a particular A-HFT and an arbitrary small aggressive order is private information, available only to the A-HFT who placed the order. Comparing the second specification above to the first isolates the effects attributable to this private information from effects attributable to public information.

Finally, I compare the additional explained performance for the specified A-HFT to the additional explained performance for all other traders. Intuitively, we want to verify that the A-HFT’s exploratory information provides extra explanatory power for the subsequent performance of trader privy to that information (the A-HFT), but not for the performance of traders who aren’t privy to it (everyone else). Note that “everyone else” includes the A-HFTs other than the specified A-HFT.

Some A-HFT accounts and B-HFT/non-HFT accounts belong to the same firms, and various B-HFTs/non-HFTs may be either directly informed or able to make educated inferences about what one or more A-HFTs do. As a result, we should not necessarily expect exploratory information generated by an A-HFT’s small orders to provide no explanatory power whatsoever for all other traders’ performance. However, we should still expect the additional explanatory power for the A-HFT’s performance to significantly exceed that for the other traders’ performance.

### 1.5.2 Empirical Implementation

Define an aggressive order to be “small” if that order’s submitted size is less than or equal to a specified size parameter, which I denote by  $\bar{q}$ .

### 1.5.2.1 A Simple Measure of Market Response

I characterize the market response to a small aggressive order using subsequent changes in orderbook depth. I examine the interval starting immediately after the arrival of a given small aggressive order and ending immediately before the arrival of the next aggressive order (which may or may not be small), and I sum the changes in depth at the best bid and best ask that occur during this interval. As in section 1.4.3, I treat sell depth as negative and buy depth as positive. I also normalize these depth changes by the sign of the preceding small aggressive order to standardize across buy orders and sell orders.

To simplify the analysis and stack the deck against finding significant results, I initially focus only on the sign of the direction-normalized depth changes. Note the direct analogy to the two-liquidity-state setting of the exploratory trading model in section 1.2.

For a given value of  $\bar{q}$ , I construct the indicator variable  $\Omega$ , with  $k$ th element  $\Omega_k$  defined by

$$\Omega_k = \begin{cases} 1 & \text{if } DC(k; any, \bar{q}) > 0 \\ 0 & \text{otherwise} \end{cases}$$

where  $DC(k; any, \bar{q})$  denotes the direction-normalized depth change following the last small aggressive order (submitted by anyone) that arrived before the  $k$ th aggressive order. Similarly, I construct the indicator variable  $\Omega^A$ , with  $k$ th element  $\Omega_k^A$  defined by

$$\Omega_k^A = \begin{cases} 1 & \text{if } DC(k; AHFT, \bar{q}) > 0 \\ 0 & \text{otherwise} \end{cases}$$

where  $DC(k; AHFT, \bar{q})$  denotes the direction-normalized depth change following the last small aggressive order *submitted by a specified A-HFT* that arrived before the  $k$ th aggressive order.

### 1.5.2.2 Estimation Procedure

In the model of exploratory trading presented earlier, exploratory information was valuable only in conjunction with information about future aggressive order flow. Following this result, I incorporate market-response information by using the indicators  $\Omega$  and  $\Omega^A$  to partition the benchmark regression from section 1.4.3.

Recall that in section 1.4.3, I estimated the equation

$$y_k = z_{k-1}\Gamma + \epsilon_k$$

where  $y_k$  denoted the cumulative price-change between the aggressive orders  $k$  and  $k + 50$ , and the vector  $z_{k-1}$  consisted of changes in resting depth between aggressive orders  $k - 1$  and  $k$ , in addition to the signs and signed executed quantities of aggressive orders  $k - 1$  through  $k - 4$ . Using the indicator  $\Omega$ , I now partition the equation above into two pieces and estimate the equation

$$y_k = \Omega_k z_{k-1} \Gamma^a + (1 - \Omega_k) z_{k-1} \Gamma^b + \epsilon_k \quad (1.9)$$

Next, I use the indicator  $\Omega^A$  to further partition (1.9), and I estimate the equation

$$\begin{aligned} y_k = & \Omega_k^A(k) \left( \Omega_k z_{k-1} \Gamma^c + (1 - \Omega_k) z_{k-1} \Gamma^d \right) + \\ & (1 - \Omega_k^A) \left( \Omega_k z_{k-1} \Gamma^e + (1 - \Omega_k) z_{k-1} \Gamma^f \right) + \epsilon_k \end{aligned} \quad (1.10)$$

The variables  $y_k$  and  $z_{k-1}$  denote the same quantities as before, and the  $\Gamma^j$  terms each represent vectors of 14 coefficients.

I estimate (1.9) and (1.10) for  $\bar{q} = 1, 5, 10, 15, 20$ , and for each specification I calculate the relative excess performance of the specified A-HFT, and of all other trading accounts on aggressive orders of size strictly greater than  $\bar{q}$ . As in section 1.4.3, I compute the performance of aggressive order  $k$  in excess of that explained by each regression by normalizing the  $k$ th residual from the regression by the sign of the  $k$ th aggressive order. I now also control for order-size effects directly by regressing the direction-normalized residuals (for the orders of size



strictly greater than  $\bar{q}$ ) on the (unsigned) executed quantities and a constant, then subtracting off the executed quantity multiplied by its estimated regression coefficient. Controlling for size effects in this manner makes results more comparable for different choices of  $\bar{q}$ . Size effects can be addressed by other means with negligible impact on the final results.

For each aggressive order larger than  $\bar{q}$  placed by the A-HFT under consideration, I compute the additional component of performance explained by (1.10) relative to (1.9) by subtracting the order's excess performance over (1.10) from its excess performance over (1.9); I stack these additional explained components in a vector that I denote by  $\Xi_A$ . I repeat this procedure to obtain the analogous vector for everyone else,  $\Xi_{ee}$ . Specification (1.10) has more free parameters than (1.9), but additional explanatory power of (1.10) due exclusively to the extra degrees of freedom will manifest equally, in expectation, for all traders, so the extra degrees of freedom alone should not cause  $\Xi_A$  and  $\Xi_{ee}$  to differ significantly.

### 1.5.3 Results

I evaluate the empirical predictions of the exploratory trading model by comparing the additional explained component of performance for each A-HFT to the additional explained component of performance for all other traders. Both predictions of the exploratory trading model are borne out in these results. Using information about the market activity immediately following an A-HFT's smallest aggressive orders (in the form of  $\Omega^A$ ) improves our ability to explain that A-HFT's performance on larger aggressive orders by a highly significant margin, relative to using only information about the activity following any small aggressive order (in the form of  $\Omega$ ). By contrast, relative to using  $\Omega$  alone, incorporating the information in  $\Omega^A$  provides little or no significant additional explanatory power for other traders' performance on larger aggressive orders.

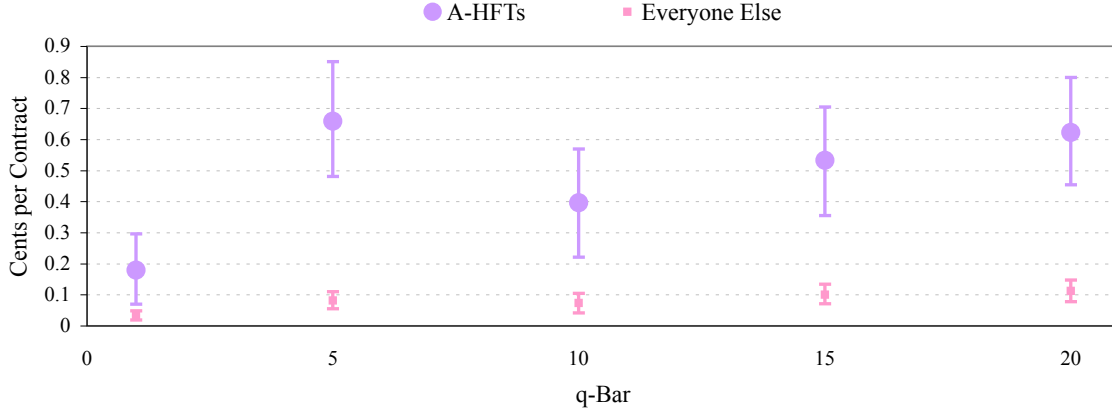


Figure 1.3: Additional Performance Explained (95% Confidence Intervals)

Figure 1.3 and Table 1.4 present the cross-sectional means of  $\Xi_A$  and  $\Xi_{ee}$  for different values of  $\bar{q}$ . Table 1.4 presents cross-sectional averages of the estimated additional gross earnings per contract on aggressive orders of submitted size greater than  $\bar{q}$  explained by regression (1.10) in excess of that explained by regression (1.9). The extra explanatory power of (1.10) reflects the contribution from the private component of information (available to the A-HFT under consideration) manifested in  $\Omega^A$ . Numbers reported for the A-HFTs are averages over the estimates for individual A-HFTs. The membership of “everyone else” depends upon the particular A-HFT being excluded, and the numbers reported for “everyone else” are averages over these slightly different groups. Units are cents per contract, and confidence intervals are constructed by bootstrap.

As a more formal comparison of the gain in explanatory power for the A-HFTs relative to the gain for everyone else, I construct 95% bootstrap confidence intervals for the difference of the pooled means  $Mean(\Xi_A) - Mean(\Xi_{ee})$ , for  $\bar{q} = 1, 5, 10, 15, 20$ . Figure 1.4 summarizes these results, which confirm what the preceding results suggested: the extra component of A-HFTs’ performance on large aggressive orders explained by using  $\Omega^A$  in addition to  $\Omega$  is significantly greater than the extra component explained for other traders.

Although the extra explanatory power for an average individual A-HFT is significantly greater than that for all other traders, the amount of performance to be explained is also somewhat greater. Comparing extra explanatory power for an individual A-HFT to extra

Table 1.4: Additional Explained Performance (Cents per Contract)

	Point Estimate		95% Confidence Interval	
	A-HFTs	Everyone Else	A-HFTs	Everyone Else
$\bar{q} = 1$	0.179	0.034	(0.069, 0.296)	(0.018, 0.048)
$\bar{q} = 5$	0.659	0.082	(0.480, 0.850)	(0.055, 0.109)
$\bar{q} = 10$	0.397	0.074	(0.221, 0.570)	(0.041, 0.104)
$\bar{q} = 15$	0.533	0.101	(0.355, 0.705)	(0.071, 0.133)
$\bar{q} = 20$	0.623	0.113	(0.453, 0.799)	(0.077, 0.147)

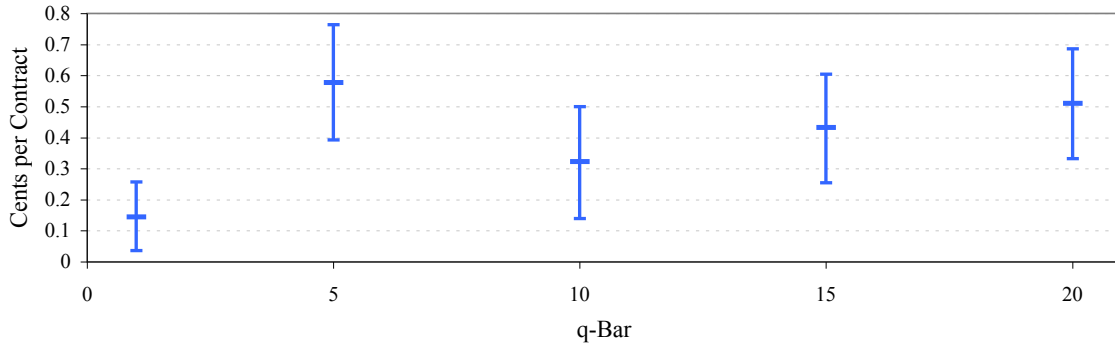


Figure 1.4: [A-HFT Addt'l Explained] - [Everyone Else Addt'l Explained] (95% CIs)

explanatory power for the complementary set of HFTs mitigates this difference. Consistent with the notion that certain B-HFTs may know something about what various A-HFTs are doing, the extra component of performance explained by using  $\Omega^A$  in addition to  $\Omega$  is larger for the complementary set of HFTs than it is for the broader “everyone except the A-HFT of interest” group. Nevertheless, aside from the case of  $\bar{q} = 1$ , the average extra explanatory power for an individual A-HFT is still significantly greater than is that for the complementary set of HFTs, as shown in Figure 1.5.

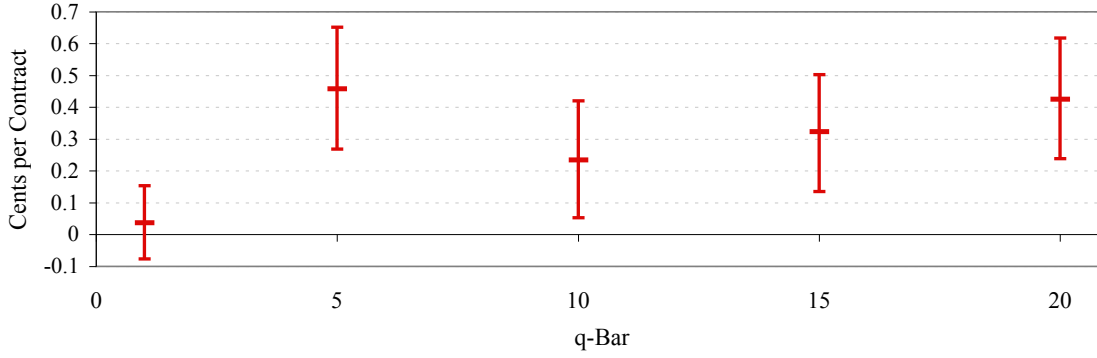


Figure 1.5: [A-HFT Addt'l Explained] - [Other HFTs Addt'l Explained] (95% Conf. Intervals)

Table 1.5 displays the numerical values from Figures 1.4 and 1.5, namely cross-sectional averages of the difference in mean additional gross earnings per contract on aggressive orders of submitted size greater than  $\bar{q}$  explained by regression (1.10) in excess of that explained by regression (1.9), between the indicated groups. Units are cents per contract, and confidence intervals are constructed by bootstrap.

Table 1.5: Additional Explained Performance for A-HFTs vs. Other Groups

	Point Estimate		95% Confidence Interval	
	A-HFTs vs. Everyone Else	A-HFTs vs. Other HFTs	A-HFTs vs. Everyone Else	A-HFTs vs. Other HFTs
$\bar{q} = 1$	0.145	0.038	(0.036, 0.257)	(-0.076, 0.154)
$\bar{q} = 5$	0.577	0.458	(0.393, 0.764)	(0.269, 0.651)
$\bar{q} = 10$	0.323	0.235	(0.139, 0.500)	(0.053, 0.420)
$\bar{q} = 15$	0.432	0.323	(0.254, 0.604)	(0.135, 0.503)
$\bar{q} = 20$	0.510	0.426	(0.332, 0.686)	(0.238, 0.618)

#### 1.5.4 Incidence of A-HFTs' Larger Aggressive Orders

As suggested by the remarks at the end of section 1.2.2.2, the prediction that exploratory information explains a significant additional component of the A-HFTs' performance tacitly requires exploratory information to help explain the incidence of the A-HFTs' larger aggressive orders. In particular, all else being equal, the exploratory trading model predicts that an A-HFT will have a greater tendency to place large aggressive orders when  $\Omega^A = 1$  than when  $\Omega^A = 0$ . A direct test of this prediction about the incidence of the A-HFTs' larger aggressive orders offers a robustness check on the results in subsection 1.5.3.

Much as the HFT in the model from section 1.2 considered the signal of future aggressive order-flow as well as the liquidity state, A-HFTs consider public market data as well as exploratory information to decide when to place large aggressive orders. The size and direction of A-HFTs' aggressive orders depend on the same variables that forecast price movements, or equivalently on the forecasts of price movements themselves. On average, the signed quantity of an A-HFT's aggressive order should be an increasing function of the future price-change expected on the basis of public information. In this context, the exploratory trading model predicts that the expected future price-change will have a larger effect on the signed quantity of an A-HFT's aggressive orders when  $\Omega^A = 1$  than it will when  $\Omega^A = 0$ .

To test the exploratory trading model's prediction about the incidence of A-HFTs' larger aggressive orders, I regress the signed quantities of a given A-HFT's aggressive orders on the associated fitted values of  $y$  from equation (1.9), partitioned by  $\Omega^A$ . In other words, for a specified A-HFT and a given value of  $\bar{q}$ , I estimate the equation

$$q_k = \beta_0 (1 - \Omega_k^A) \hat{y}_k + \beta_1 \Omega_k^A \hat{y}_k + \epsilon_k \quad (1.11)$$

where  $q_k$  denotes the signed submitted quantity of the A-HFT's  $k$ th aggressive order,  $\hat{y}_k$  denotes the relevant fitted value of  $y_k$  from the public-information regression (1.9), and  $\Omega^A$  is the usual indicator function. I restrict the  $\beta$  coefficients to be the same across all A-HFTs.

Table 1.6 displays the coefficient estimates from (1.11) for various values of  $\bar{q}$ . A Wald test

rejects the null hypothesis  $\beta_0 = \beta_1$  at the  $10^{-15}$  level for all values of  $\bar{q}$ . As the exploratory trading model predicts, holding fixed the price-change expected on the basis of public information, the average A-HFT places significantly larger aggressive orders when  $\Omega^A = 1$  than when  $\Omega^A = 0$ .

Table 1.6: Differential Effects of Predicted Price-Changes on A-HFT Signed Order Size

	Point Estimates		Standard Errors	
	$\beta_0$	$\beta_1$	$\beta_0$	$\beta_1$
$\bar{q} = 1$	13.35	15.26	0.094	0.162
$\bar{q} = 5$	13.41	15.11	0.093	0.169
$\bar{q} = 10$	13.42	14.97	0.095	0.160
$\bar{q} = 15$	13.34	15.10	0.095	0.159
$\bar{q} = 20$	13.23	15.30	0.094	0.160

## 1.6 Practical Significance of Exploratory Information

The losses that A-HFTs incur on their small aggressive orders offer a natural point of comparison for the gains that can be explained from the information generated by those orders. Table 1.7 displays the additional component of A-HFTs' profits on large aggressive orders directly explained using exploratory information (in the form of  $\Omega^A$ ) as a percentage of A-HFTs' losses on small aggressive orders. In one sense, given the extreme simplicity and coarseness of the  $\Omega$ -operators as representations of exploratory information, the results in Table 1.7 suggest gains that are surprisingly large in practical terms. At the same time, the gains from exploration should at least weakly exceed the costs, and the additional gains directly explained using  $\Omega^A$  fall short of this mark.

Representations of exploratory information richer than  $\Omega^A$  are extremely easy to con-

Table 1.7: Extra Explained Gains on Large AOs vs. Losses on Small AOs

	$(Extra\ Explained\ Gains) /  Losses $
$\bar{q} = 1$	33.05%
$\bar{q} = 5$	11.27%
$\bar{q} = 10$	4.48%
$\bar{q} = 15$	5.39%
$\bar{q} = 20$	4.79%

struct. For example, an obvious extension would be to consider the not only the sign, but also the *magnitude* of the direction-normalized depth change following an exploratory order. Regardless of the particular representation of exploratory information used, though, the additional explained component of A-HFTs' profits on the aggressive orders they place is likely to understate the true gains from exploration. As the simple model in section 1.2 illustrates, exploratory information is valuable in large part because it enables a trader to avoid placing unprofitable aggressive orders. However, estimates of the additional explained component of profits on A-HFTs' aggressive orders necessarily omit the effects of such avoided losses. While this bias, if anything, makes the preceding findings of statistical significance all the more compelling, it also complicates the task of properly determining the practical importance of exploratory information.

### 1.6.1 Simulated Trading Strategies

To investigate the gains from exploratory information, including the gains from avoiding unprofitable aggressive orders, I examine the effects of incorporating market-response information from small aggressive orders into simulated trading strategies. The key advantage of working with these simulated trading strategies is that avoided unprofitable aggressive orders can be observed directly.

The basic trading strategy that I consider is a simple adaptation of the benchmark regres-

sion from section 1.4.3. I specify a threshold value, and the strategy entails nothing more than placing an aggressive order with the same sign as  $\hat{y}_k$  whenever  $|\hat{y}_k|$  exceeds that threshold. To make this strategy feasible (in the sense of using only information available before time  $t$  to determine the time- $t$  action) I compute the forecast of future price movement,  $\hat{y}_k$ , using the regression coefficients estimated from the previous day's data. I incorporate market-response information into this strategy by modifying the rule for placing aggressive orders to, "place an aggressive order (with the same sign as  $\hat{y}_k$ ) if and only if all three of the following conditions hold:

- $|\hat{y}_k|$  exceeds its specified threshold,
- The direction-normalized depth-change following the last small aggressive order (placed by anyone) exceeds a specified threshold, and
- The direction-normalized depth-change following the last small aggressive order *placed by an A-HFT* exceeds a (possibly different) specified threshold."

Choosing a threshold of  $-\infty$  will effectively remove any of these conditions.

Each strategy yields a set of times to place aggressive orders, and the associated direction for each order. To measure the performance of a given strategy, I compute the average profitability of the indicated orders in the usual manner, with the assumption that these aggressive orders are all of a uniform size.

Relative to A-HFTs' losses on small aggressive orders, the additional component of A-HFTs' profits directly explained using  $\Omega^A$  is smallest when  $\bar{q} = 10$ , and I present results for  $\bar{q} = 10$  to highlight the impact of accounting for avoided losses on estimates of the gains from exploratory information. Results for other values of  $\bar{q}$  are similar.

### 1.6.1.1 Three Specific Strategies

All three threshold parameters affect strategy performance, so to emphasize the role of market-response information, I present results with the threshold for  $|\hat{y}_k|$  held fixed. Varying the threshold for  $|\hat{y}_k|$  does not alter the qualitative results. In particular, it is not possible to



achieve the same gains in performance that result from incorporating exploratory information by merely raising the threshold for  $|\hat{y}_k|$ . The forecast  $\hat{y}_k$  uses coefficients estimated from the previous day's data, and these forecasts exhibit increasing bias as the  $z_{k-1}$  observations assume more extreme values.

I consider a range of threshold values for the direction-normalized depth-change following the last small aggressive order placed by anyone, but, for expositional clarity, I present results for three illustrative threshold choices for the direction-normalized depth-change following the last small aggressive order placed by an A-HFT. Specifically, I consider thresholds of  $-\infty$  (no A-HFT market-response information), 0 (the same information contained in  $\Omega^A$ ), and 417 (the 99th percentile value). Figure 1.6 displays the performance of these three strategies over a range of threshold values for the market response following arbitrary small aggressive orders.

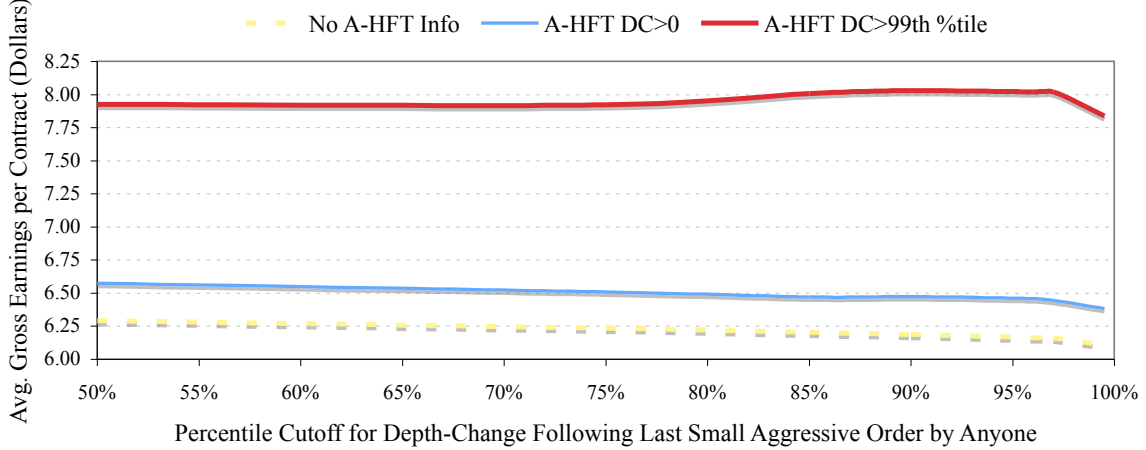


Figure 1.6: Absolute Gains from Exploratory Information

While the performance gains from incorporating A-HFT exploratory information are obvious, an equally important feature of the results above is more subtle. The A-HFTs' average gross earnings on aggressive orders over size 10 of \$7.78 per contract are well above the peak performance of the strategy that uses only public information, but substantially below the performance of the strategy that incorporates the A-HFTs' exploratory information with the higher threshold. This is exactly the pattern that we should expect, given that the former strategy excludes information that is available to the A-HFTs and the latter strategy includes

information that is not available to any individual A-HFT, so these results help to confirm the relevance and validity of this simulation methodology.

### 1.6.1.2 Gains from Exploration Relative to Losses on Exploratory Orders

Although the two strategies that incorporate exploratory information from the A-HFTs' small aggressive orders outperform the strategy that does not, the orders that generated the exploratory information were costly. To compare the gains from this exploratory information to the costs of acquiring it, I first multiply the increases in per-contract earnings for the two exploratory strategies (scaled by the respective number of orders relative to the public-information strategy) by the A-HFTs' combined aggressive volume on orders over size 10.<sup>13</sup> I then divide these calibrated gains by the A-HFTs' actual losses on aggressive orders size 10 and under. The resulting ratio is the direct analogue of the percentages in Table 1.7.

Figure 1.7 displays the calibrated ratio of additional gains to losses for each exploratory simulated strategy over a range of threshold values for the market response following arbitrary small aggressive orders.

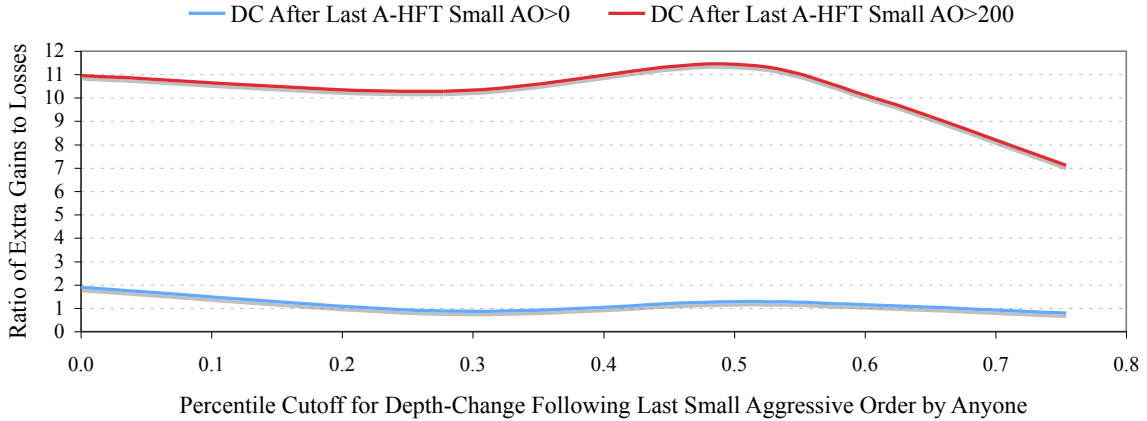


Figure 1.7: Gains from A-HFT Exploratory Info Relative to Losses on Exploratory Orders

Using information from the A-HFTs' exploratory orders analogous to that in  $\Omega^A$ , the additional gains are roughly 15% larger than the losses on exploratory orders. Whereas

<sup>13</sup>The two strategies that incorporate exploratory information select subsets of the aggressive order placement times generated by the public-information-only strategy. Although the selected orders tend to be more profitable, they are also fewer in number.

the extra component of the A-HFTs' performance directly explained using  $\Omega^A$  represented less than 5% of A-HFTs' losses on exploratory orders, the analogous estimated performance increases more than offset the costs of exploration once we include the gains from avoiding unprofitable aggressive orders. In the case of the strategy that employs information from the A-HFTs' exploratory orders with the higher threshold, the estimated gains from exploration exceed the costs by more than one-third.

## 1.7 Discussion

### 1.7.1 Broader Scope for Exploratory Gains from Aggressive Orders

The empirical results in the preceding sections focused on the information generated by the A-HFTs' smallest aggressive orders. While their otherwise-perplexing unprofitability made these orders the most obvious starting point for an empirical study of exploratory trading, there is no theoretical reason why these small orders should be the sole source of exploratory information. In the baseline exploratory trading model, it was only to highlight the key aspects of the model that I assumed the HFT's period-1 order was expected to lose money and served no purpose other than exploration.

In principle, even aggressive orders that an A-HFT expects to be directly profitable could produce valuable, private, exploratory information. To investigate this possibility, I repeat the analysis of section 1.5.2.2 setting  $\bar{q} = 25, 30, 35, 40, 45, 50, 60, 75, 90$ . The A-HFTs' incremental aggressive orders included with each increase of  $\bar{q}$  beyond  $\bar{q} = 20$  are directly profitable on average, and yet the market response following these orders still provides significantly more additional explanatory power for the A-HFTs' performance on larger aggressive orders than it provides for that of other traders. Indeed, the additional explained components of the A-HFTs' performance are markedly larger than those for  $\bar{q} = 1, 5, \dots, 20$ ; see Figure 1.8.

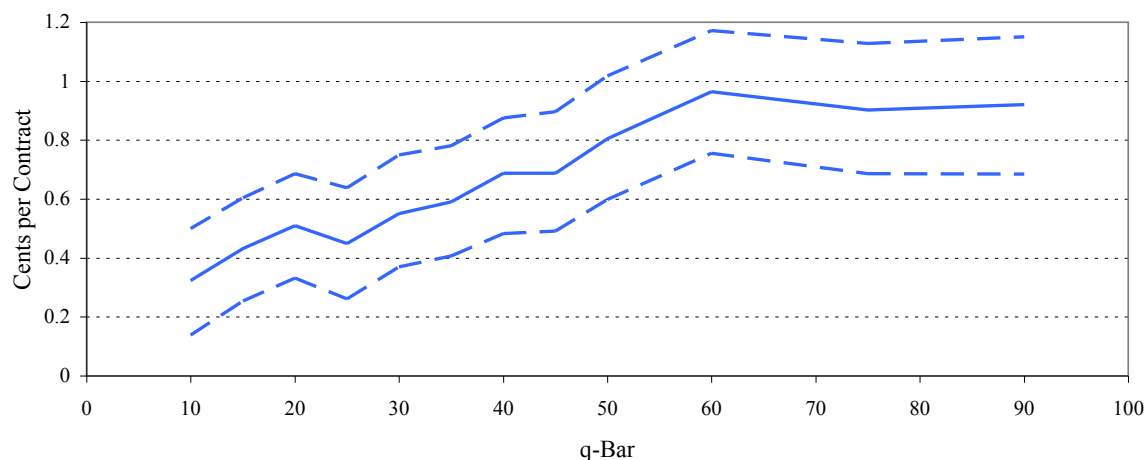


Figure 1.8: [A-HFT Addt'l Explained] - [Everyone Else Addt'l Explained] (95% CIs)

## 1.7.2 Exploratory Trading and Speed

Further analysis of the exploratory trading model reveals natural connections between exploration and two important concepts of speed.

### 1.7.2.1 Low Latency

One common measure of trading speed is *latency*—the amount of time required for messages to pass back and forth between a trader and the market. While low-latency operation and high-frequency trading are not equivalent, minimal latency is nonetheless a hallmark of high-frequency traders. For a trader who can identify profitable trading opportunities, there is obvious value to possessing latency low enough to take advantage of these opportunities before they disappear. The new insight from the exploratory trading model concerns the more subtle matter of how low latency connects to the identification of such opportunities.

In the model of exploratory trading developed in section 1.2, the HFT's inference about  $\Lambda$  on the basis of market activity following his aggressive order in period 1 implicitly depends on a notion related to latency. If we suppose that random noise perturbs the orderbook, say according to a Poisson arrival process, then the amount of noise present in the HFT's observation of the market response in some interval following his aggressive order will depend

on the duration of that interval. The duration of this interval will depend in large part upon the rate at which market data is collected and disseminated to the HFT, that is, the “temporal resolution” of the HFT’s data. Although this temporal resolution does not directly depend on the HFT’s latency, the HFT’s latency *is* implicitly constrained by the temporal resolution of his market information.

The finer temporal resolution required for low-latency operation enables low-latency traders to obtain meaningful—and empirically valuable—information about the market activity immediately following their aggressive orders, and this information degrades at coarser temporal resolutions. The empirical results from section 1.5.3 provide a concrete illustration of this effect. The changes in resting inside depth immediately following an arbitrary aggressive order are less useful for forecasting price movements than are the analogous changes following an A-HFT’s aggressive order, but the two can only be distinguished (by the A-HFT) in data with a sufficient level of temporal disaggregation.

### 1.7.2.2 High Frequency

Exploratory trading bears a natural relationship to the practice of placing large numbers of aggressive orders—what might be considered “high-frequency trading” in the most literal sense.

Exploratory information generated by a given aggressive order is only valuable to the extent that it can be used to improve subsequent trading performance. Because exploratory information remains relevant for only some finite period, the value of exploratory information diminishes as the average interval between a trader’s orders lengthens. The exploratory trading model readily captures this effect if we relax the simplifying assumption that the liquidity state  $\Lambda$  remains the same between periods 1 and 2. Suppose that  $\Lambda$  evolves according to a Markov process, such that with probability  $\tau$ , a second  $\Lambda$  is drawn in period 2 (from the same distribution as in period 1), and with probability  $1 - \tau$ , the original value from period 1 persists in period 2. Intuitively,  $\tau$  parametrizes the length of period 1, and this length increases from zero to infinity as  $\tau$  increases from zero to unity. As  $\tau$  tends towards unity—i.e., as the

length of period 1 increases to infinity—the liquidity state in period 1 becomes progressively less informative about the liquidity state in period 2.

As discussed in section 1.7.1, both theory and empirical evidence suggest that almost any aggressive order that a trader places generates some amount of exploratory information. Consequently, as a trader places aggressive orders in greater numbers, he will gain access to greater amounts of exploratory information. Furthermore, the average time interval between a trader’s aggressive orders necessarily shrinks as the number of those orders grows, so the exploratory information produced by each order tends to become more valuable to the trader. These synergistic effects dramatically magnify the potential gains from exploratory information for traders who place large numbers of aggressive orders.

#### **1.7.2.3 Latency Détente**

There has been much speculation about HFTs engaging in an “arms race” for ever-faster processing and ever-lower latency. If high-frequency trading entailed nothing more than reacting to publicly observable trading opportunities before anyone else, HFTs would indeed face nearly unbounded incentives to be faster than their competitors. While reaction speed is certainly one dimension along which HFTs compete, the empirical evidence of exploratory trading suggests that the A-HFTs, at least, can also compete along another dimension—exploration. Since exploratory trading provides the A-HFTs with private information, a trader who uses only public information will not necessarily be able to dominate the A-HFTs, even if that trader is faster than every A-HFT. Similarly, an A-HFT could potentially compensate for having (slightly) slower reactions than the other A-HFTs by engaging in greater levels of exploration.

#### **1.7.3 Beyond A-HFTs: Other HFTs and Other Markets**

Exploratory trading is not universally relevant to all HFT activity in all markets. Equities markets, for instance, may not exhibit the predictability in demand that makes exploratory trading viable in the E-mini market, so HFTs in these markets might primarily concern them-

selves with obtaining superior forecasts of demand, or they might employ some completely different technique. However, exploratory trading in the E-mini market depends only on the market's structure and aggregate dynamics; it does not depend directly on any specific features of the E-mini contract. The prevalence of exploratory trading in other markets is ultimately an empirical matter, but markets similar to the E-mini in size and structure could easily support exploratory trading.

Even in the E-mini market, an important component of HFT activity lies outside the immediate province of the exploratory trading model. Nevertheless, the scope for exploratory trading extends well beyond the aggressive activity of A-HFTs considered thus far. Though I have focused on the A-HFTs up to this point, the B-HFTs could also reap exploratory rewards from their aggressive orders, as could potentially any trader with similar capabilities. The B-HFTs' overall performance on aggressive orders does not present the same ostensible affront to market efficiency as does that of the A-HFTs, but the B-HFTs' aggressive orders nonetheless outperform both those of non-HFTs, and the baseline econometric benchmark, by a wide margin. If inventory management or risk-control considerations force B-HFTs to place unprofitable aggressive orders, exploratory trading could help to explain how the B-HFTs mitigate the associated losses.

Alternatively, if nothing forces the B-HFTs to place aggressive orders, then the B-HFTs' consistent losses from aggressive trading are puzzling in their own right, much as the A-HFTs' losses on small aggressive orders were. Although the B-HFTs do not recoup their losses on other aggressive orders as do the A-HFTs, they make enough from their passive trading to earn positive profits overall. Passive trading strategies, just like aggressive ones, would benefit from the superior price forecasting that exploratory information makes possible, so exploratory trading could help to explain the activity of B-HFTs in this scenario as well.<sup>14</sup>

---

<sup>14</sup>Total trading profits from any transaction net to zero, so if a trader earns money on an aggressive order, his passive counter-party loses money. Since exploratory information is valuable to an aggressor, it follows immediately that it is also valuable to a passor.

## 1.8 Conclusion

Empirical evidence strongly suggests that the concept of exploratory trading developed in this paper helps to explain the mechanism underlying certain HFTs' superior capacity to profitably anticipate price movements in the E-mini market. The exploratory trading model also illuminates the manner in which these HFTs benefit from low-latency capabilities and from their submission of large numbers of aggressive orders.

Exploratory trading is a form of costly information acquisition, albeit an unfamiliar one. HFTs who engage in exploratory trading are doing something more than merely reacting to public information sooner than other market participants. This raises the possibility that HFTs, through exploratory trading, uniquely contribute to the process of efficient price discovery. However, exploratory trading differs from traditional costly information acquisition in several important respects. First, the information that exploratory trading generates does not relate directly to the traded asset's fundamental value, but rather pertains to unobservable aspects of market conditions that could eventually become public, *ex-post*, through ordinary market interactions. Also, because exploratory trading operates through the market mechanism itself, exploration exerts direct effects on the market, distinct from the subsequent effects of the information that it generates.

Finally, since HFTs appear to trade ahead of predictable demand innovations—albeit in a sophisticatedly selective manner—the research of De Long *et al.* (1990) potentially suggests that HFTs could have a destabilizing influence on prices if suitable positive-feedback mechanisms exist.

Comprehensive analysis of the theoretical and empirical aspects of these myriad issues lies beyond the scope of this essay, but the theory and evidence presented herein provide a starting point from which to rigorously address the market-quality implications of high-frequency trading going forward.



## Chapter 2

# Foundations of the Disposition Effect: Experimental Evidence<sup>15</sup>

### 2.1 Introduction

Of the many unresolved puzzles about trading in financial markets, perhaps the most widely discussed is the so-called “disposition effect”—the robust finding for many assets that investors have a greater propensity to sell the asset at a gain relative to its purchase price than they have to sell the asset at a loss. In their seminal 1985 paper, Shefrin and Statman argued that the psychological elements of Kahneman and Tversky’s (1979) “prospect theory” would work together with some other factors to induce a disposition effect among investors.

Odean (1998) analyzed the trading records from 10,000 individual accounts at a large discount brokerage. This detailed, individual-level data allowed Odean to establish near-incontrovertible evidence of the disposition effect’s existence, as well as strong evidence that standard mechanisms, such as portfolio rebalancing, tax-loss selling or belief in mean reversion, could only account for a fraction of the observed effect. While Odean’s results, as well as subsequent analyses, provide evidence against classical explanations of the disposition effect, these findings do not provide direct evidence that prospect theory is the proper explanation.

---

<sup>15</sup>Co-authored with Johanna Möllerström

Nevertheless, an explanation of the disposition effect based on an informal appeal to prospect theory has gained widespread acceptance in the economics and finance literature.

### 2.1.1 Prospect Theory

As demonstrated repeatedly in the decision-theory and psychology literatures, the traditional “expected utility” framework is not a fully realistic explanation of how people make decisions involving risk. Prospect theory is a descriptive model of choice under uncertainty that draws upon experimental evidence to refine and correct many predictions of expected utility theory. Prospect theory maintains the basic formal structure familiar from an expected utility context, wherein an agent’s preferences can be expressed as the inner product of some function evaluated in each possible state of the world, with some other function evaluated at the *ex-ante* probability of the appropriate state. In other words, in an expected-utility context, we characterize an agent’s preferences with

$$\int_{\omega \in \Omega} U(x(\omega)) [1 * p(\omega)] d\omega \quad (2.1)$$

where  $p(\omega)$  is the probability for state  $\omega$ , and  $U(\cdot)$  is a standard utility function. Analogously, in a prospect-theoretic context we characterize an agent’s preferences with

$$\int_{\omega \in \Omega} v(x(\omega)) \pi(p(\omega)) d\omega \quad (2.2)$$

The functions  $v(\cdot)$  and  $\pi(\cdot)$  are known, respectively, as the “value function” and the “probability weighting function.” While the probability weighting function has proven important elsewhere, prospect-theoretic explanations of the disposition effect have focused almost exclusively on the value function.

Whereas standard expected utility theory considers the total endowment that an agent would have in various possible states of the world (e.g.,  $U(W + x(\omega))$ , where  $W$  denotes total wealth), prospect theory considers the agent’s (state-wise) gains/losses relative to some reference point (e.g.,  $v(x(\omega))$ ). Furthermore,  $v(\cdot)$  has the following properties:

1.  $v(0) = 0$  . This is just a convention setting the reference point to the origin.
2.  $\lim_{\epsilon \rightarrow 0} (v'(-\epsilon) - v'(\epsilon)) > 0$ . The value function  $v(\cdot)$  is not everywhere differentiable; there is a “kink” at the origin, and the agent is more sensitive to small losses than to small gains.
3.  $v''(x) \begin{cases} < 0 & x > 0 \\ > 0 & x < 0 \end{cases}$  The value function  $v(\cdot)$  is concave over gains, but *convex* over losses

As an example, Barberis and Xiong (2009) consider the value function

$$v(x) = \begin{cases} x^\alpha & x \geq 0 \\ -\lambda(-x)^\alpha & x < 0 \end{cases} \quad \alpha \in (0, 1), \lambda > 1$$

Following the experimental estimates of Kahneman and Tversky (1992), Barberis and Xiong use as baseline parameter values  $\alpha = 0.88$  and  $\lambda = 2.25$ .

### 2.1.2 Explaining (?) the Disposition Effect

The standard explanation of how prospect-theoretic preferences generate a disposition effect abstracts away from the initial purchase decision, and considers a setting with a single stock, taking the purchase price of the stock to be the reference point. As the price of the stock increases above the initial purchase price, the investor moves into the “gains” region of her value function. Since the value function is concave over this region, the investor will eventually reduce or even exit her position in the stock because she will reach a point where the benefit that she expects to accrue from further gains is smaller than the detriment that she would be caused by a decrease in her gains. By analogous reasoning, the investor’s desire to hold the stock increases as the price of the stock decreases below the initial purchase price and the investor moves into the convex “losses” region of her value function.<sup>16</sup>

---

<sup>16</sup>We could reach similar conclusions by appealing purely to the “loss aversion” property of the value function (i.e.,  $\lim_{\epsilon \rightarrow 0} (v'(-\epsilon) - v'(\epsilon)) > 0$ ), but the arguments are more subtle, less robust, and lead to additional testable predictions that our experimental results do not support. We will discuss this point in subsection 2.5.2.

Despite the intuitive appeal of the informal arguments sketched above, recent theoretical research demonstrates that prospect theory often does not suffice to induce a disposition effect. Under formal scrutiny, the standard informal arguments frequently break down. For example, Barberis and Xiong (2009) find that the ability of prospect theory to predict a disposition effect is very sensitive to the choice of reference point, the timing of evaluation and the general calibration. Empirical findings by Rangelova (2000), Calvet *et al.* (2009), and more recently Ben-David and Hirshleifer (2012) challenge the prospect-theoretic-preference explanation of the disposition effect. Chapter 3 of this dissertation constructs a fully rational, parsimonious model in which the standard disposition effect—as well as the numerous observed departures from the standard disposition effect are explained by minor informational frictions. While the model in chapter 3 does not require additional behavioral/psychological mechanisms, neither does it necessarily rule such mechanisms out. Research in the vein of Ben-David and Hirshleifer, or Rangelova, points towards a role for investor beliefs in generating the disposition effect, and recent research such as that of Barberis and Xiong (2009) has begun to weaken the most common prospect-theory explanation, but there remains considerable scope for other pseudo-behavioral mechanisms to be at work.

When Shefrin and Statman first proposed that psychological mechanisms would generate a disposition effect, they invoked not only prospect-theoretic preferences, but also other mechanisms, including a desire to avoid admitting mistakes and avoid the associated feelings of regret. Extensive research in the field of social psychology, e.g., Zeelenberg *et al.* (1996), suggests that the desire to minimize regret affects individuals' decision-making behavior. We use a laboratory experiment to explicitly study the role that “regret aversion” plays in generating the disposition effect.<sup>17</sup>

---

<sup>17</sup>Shortly after we conducted our experiment, Chang, Solomon and Westerfield, began a related line of research. Although it is technically subsequent to our study, the work of Chang *et al.* originated independently and essentially in parallel. Furthermore, the experimental framework of Chang *et al.* is entirely different from our own, so the two studies are complementary.

### 2.1.3 Experimental Study of Regret Aversion

Our experimental design applies the empirical finding from the psychology literature (e.g., Zeelenberg *et al.*, (1998)) that a feeling of responsibility is essential for feelings of regret to arise. Drawing on this finding, we assume in our study that a subject can feel regret about an outcome only if she made a decision that potentially contributed to the outcome. Subjects in our experiment all engage in an identical task of trading simulated assets, but subjects in one treatment group are *randomly assigned* the assets they initially hold (call this initial time “period 0”), while subjects in the other treatment group get to *choose* which assets to initially hold. Subjects who were randomly assigned their initial assets holdings made no decisions that could be regretted, but the subjects who chose their initial asset holdings did make decisions that could be regretted. We compare the disposition effect across these two treatment groups to identify differences potentially attributable to some “regret aversion” mechanism.

Although the treatment where subjects choose which assets to hold initially introduces scope for regret, the “choice” treatment may also introduce another factor. The “choice” treatment requires subjects to actively think about a trading-related task, make some decisions, then implement those decisions. As we will discuss at greater length in section 2.3, subjects have no information to guide their initial choice of assets, so the task merely entails selecting three different elements from a set consisting of five *ex-ante*-identical risky assets and the one riskless asset; subjects implement their decisions by entering three numbers into their computer terminal. While low in an absolute sense, the level of subject engagement demanded by the “choice” treatment is high relative to the level of subject engagement demanded by the “assigned” treatment.

To assess and control for the effects of forced engagement, we consider an additional treatment, “writing,” designed to increase treated subjects’ engagement without directly changing anything else. In the “writing” treatment, subjects write down on paper which assets they hold in period 0 (regardless of whether the assets were chosen or assigned), and whether each asset’s price increases or decreases between period 0 and period 1. The “writing” treatment doesn’t introduce any new information or directly affect the act of trading at period 1, but it

makes treated subjects devote slightly more time and energy to looking at the available information. We partition the “choice” treatment into “choice, writing” and “choice, no writing,” and we similarly partition the “assigned” treatment into “assigned, writing” and “assigned, no writing.”

Experimentally, we find a very pronounced disposition effect, and we find strong evidence that the “writing” treatment reduces this disposition effect. Although we find no significant effect from the “choice” treatment, our experimental results suggest an important synergistic interaction between the “choice” and “writing” treatments. Auxiliary findings related to subjects’ desire (or lack of desire) to choose the riskless asset when given the opportunity also shed new light on the plausible origins of the disposition effect.

The remainder of the paper is organized as follows: Section 2.2 describes the general format of the experiment, explains why and how this setup offers a clean way to study the disposition effect, then draws on this exposition to highlight the theoretically appealing aspects of the regret-aversion mechanism; section 2.3 presents the finer details of the experimental design and implementation. Section 2.4 presents the main experimental results, first documenting evidence that subjects in our experiment do indeed exhibit a disposition effect, then subsequently examining the influence of the two treatments. Section 2.5 discusses the experimental findings regarding attention, regret aversion, and their interaction, then presents some auxiliary results related to subjects’ initial asset-allocation decisions. Section 2.6 similarly concludes.

## **2.2 Experimental Framework and Motivation**

### **2.2.1 Basic Format**

We define a sequence of 11 periods to be a “trading session” and define a collection of 12 such sessions as one run of our experiment. Across all treatment groups in our experiment, every subject ends period zero of each session with their initial endowment split equally into thirds and invested in three distinct assets. For the remainder of the session, subjects begin each of

the next 10 periods by observing the price-changes that occurred after the preceding period ended; then, once the most recent price-change has become observable, subjects may trade any or all of the six assets, A-F. Five assets, labeled A - E, are explicitly risky, while the one asset labeled F (or “cash”) is riskless, and its price remains constant.

During the experiment itself, the non-cash assets were referred to strictly as “assets A-E,” but we will call them “stocks” in the present discussion for expositional ease.

#### **2.2.1.1 Distinguishing a Disposition Effect from a Rational Response**

If the disposition effect documented in the empirical literature were readily explicable as the rational behavior of investors with standard preferences, it would merit little further study. A crucial feature of our experimental design is that within a given session, each stock’s price has a unique and constant probability of increasing each period, so that movements of a stock’s price in past periods are indicative of movements in future periods. The price processes are chosen so that if a stock’s price has increased (decreased) in the past periods of a given session, it is more likely to increase (decrease) in the rest of that session, and subjects are informed of this fact. At the end of the experiment, subjects earn a fixed percentage of the final value of their portfolio from one randomly chosen session, so subjects have an clear incentive to try to maximize the value of their respective portfolios every period. As we will further discuss, a disposition effect in the first trading period constitutes an unambiguous mistake in this setting, so our experimental design enables us to distinguish a disposition effect from rational behavior. This key feature of our design draws on the seminal the work of Weber and Camerer (1998).

#### **2.2.1.2 Price Processes**

In each period, for each asset A-E, it is first determined whether the price of an asset should rise or fall, then the magnitude of the price increment is selected independently and uniformly at random to be 1, 3, or 5 percent. During a given experimental session, each of the five stocks is associated with a unique, fixed probability of a price increase, as shown in Table 2.1 below:

Table 2.1: Price Increase Probabilities

Asset Identity	$\mathbb{P}\{\textit{price increases}\}$ in Period $t$
++	0.65
+	0.55
0	0.5
−	0.45
−−	0.35

Subjects are informed about this structure, but they do not know which stock (labeled A-E) is associated with which price-increase probability during any given session. Subjects are also informed that the A-E labels are randomly reassigned among the risky assets before the start of each session.

In each session, subjects observe five price-path histories—one for each of the five stocks (++, +, 0, −, and −−). During a given session the A-E labels are randomized separately for each subject, but up to a relabeling, all subjects observe the same five price-paths; the price change of, e.g., the ++ stock after period  $t$  of session  $s$  is the same for all subjects.

### 2.2.2 Cleanly Isolating a Disposition Effect

While Weber and Camerer’s clever “persistent price movement” device provides a useful starting point, it does not fully suffice for the purposes of our study. Whereas Weber and Camerer primarily sought to discover whether a disposition effect would manifest in an experimental setting, our objective is to study how various factors influence such a disposition effect. Since our experimental objectives demand maximal precision, we base our analysis on only the trades that subjects make in period 1 of each session. There are two main reasons for this tight focus. First, to the extent that prospect theory is one of the competing alternatives against which we wish to have some power, we want to leave as little flexibility as possi-



ble for plausible choices of reference point. More importantly, both the information set and the set of possible actions available to a subject during period 1 are small enough for fairly comprehensive analysis to be tractable.

### 2.2.2.1 The Optimality of Selling Losers

Recall that the respective magnitudes of price changes for a given asset are independent of the direction of past and future price changes for that asset. Together with the fact that price-change magnitudes are computed as percentages of an asset’s price, this implies that the period-1 rational expectation of an asset’s performance over the rest of a session depends only on the sign of that asset’s price-change between periods 0 and 1. If all five stocks’ respective prices change in the same direction between periods 0 and 1, subjects obtain no new information about which stock has which trend, but this situation happens not to arise in our experiment, so we ignore this degenerate case. When the price of at least one stock goes up, and the price of at least one stock goes down, subjects *do* obtain new information from the price movements.

Note that the expected value of the price change for a randomly chosen asset is zero, so in period 0, every stock is second-order stochastically dominated by cash. If at least one stock’s price has increased, then a stock whose price decreased (call it a “loser”) offers a lower expected return in terms of period-1 expectations than it did in terms of period-0 expectations. Since the incipient loser was already second-order stochastically dominated by cash in period 0, the loser is first-order stochastically dominated by cash in period 1.

A period-1 “winner” (i.e., a stock whose price increased between periods 0 and 1) offers a higher expected return than cash, but it’s also riskier. Subjects’ trading decisions regarding the period-1 winners which the subjects held at period 0 therefore offer little empirical traction. By contrast, rationalizing a subject’s decision to hold a first-order stochastically dominated loser rather than selling it for cash in period 1 is somewhat difficult. For precisely this reason, we focus on the proportion of losses realized in period 1. For each subject,  $i$ , we compute the

“disposition coefficient” in session  $s$  by the following equation:

$$PLR_{i,s} \equiv \frac{\# \text{ shares sold at a loss in period 1 of session } s}{\# \text{ shares of losers held at start of period 1 of session } s}$$

For completeness, we also present summary results for the proportion of gains realized in period 1, but our central analysis examines the theoretically cleaner “proportion of losses realized.”

### 2.2.2.2 Narrow Framing

Suppose that a subject has standard preferences, defined over her total wealth  $w$ , and suppose that those preferences can be represented by the utility function  $u(w)$ , where  $u' > 0$ ,  $u'' < 0$ . Let  $c$  denote the performance-based compensation that the subject earns in the experiment, and assume that  $0 \leq c \leq C$ . Provided that  $C$  is negligible relative to  $w$ , a reversal of Rabin’s (2000) calibration argument implies that  $u'(w) \approx u'(w + C)$ , whence it follows by monotonicity that  $u'(w + c)$  is approximately constant for  $c \in [0, C]$ , i.e., the subject will be approximately risk-neutral with respect to the performance-based compensation in the experiment. If this is the case, then the subject’s optimal strategy will be to invest all of her money each period in the stock(s) most likely to be the ++ stock (in the first period, every “winner” stock is equally likely to be the ++ stock).

However, subjects usually do not behave as though they are risk-neutral with respect to small payoffs in experiments. The standard mechanism invoked to reconcile this robust empirical finding with the preceding theoretical arguments is *narrow framing*. The precise incarnations vary somewhat, but the general idea of narrow framing is to suppose that subjects evaluate the gambles to which they are exposed in an experiment in isolation, rather than evaluating those gambles in terms of their aggregate wealth.

**Standard Preferences** In the notation introduced above, we could impose a form of narrow framing by assuming that a subject has preferences directly over  $c$ , so that she bases her decisions on the quantity  $u(c)$  rather than  $u(w + c)$ ; Barberis and Huang (2001) refer to this

form of narrow framing as “portfolio accounting.”

In this case, the subject faces a dynamic asset allocation problem of the form

$$\begin{aligned} & \max_{\{q_s\}_{s=0}^T} \mathbb{E}_t [u(q_T p_T)] \\ \text{s.t. } & q_{t+1} p_t = q_t p_t \quad t = 1, 2, \dots, T-1 \\ & q_0 p_0 = 120 \end{aligned}$$

where  $q_s$  is a (row) vector of asset holdings at date  $s$ , and  $p_s$  is a (column) vector of asset prices at date  $s$ .

Since the subject now cares about risk as well as expected return, it is no longer necessarily true that the subject’s optimal policy will be to invest all of her money each period in the stock(s) most likely to be the ++ stock. However, the equal-weighted portfolio of all five stocks is second-order stochastically dominated by cash, so we can conclude that the subject will hold at most four stocks, and therefore it can never be optimal for the subject to hold the stock which is most likely to be the -- stock. In the first period, all stocks that went down are equally likely to be the -- stock, so every “loser” stock in period 1 is dominated by cash. Hence, despite using narrow framing in the “portfolio accounting” sense, a subject with standard preferences will still find it optimal to sell all her losers in period 1.

**Prospect Theory Preferences** In our experimental setting, even a subject with prospect-theory preferences who uses a “portfolio accounting” version of narrow framing will find it optimal to sell all her losers in period 1.

Consider a subject with prospect-theory preferences, and by analogy to the “portfolio accounting” from the standard-preferences case, assume that the subject’s value function applies to her overall gains or losses at the end of the experimental session. The subject’s prospect-theoretic preferences will dictate the conditionally optimal fraction of wealth to devote to stock in each period, but will not directly affect the subject’s optimal allocation of wealth among different stocks. In particular, prospect-theoretic preferences defined over overall portfolio gains/losses can never induce a subject to hold a portfolio of stocks that has

a strictly lower expected return than a feasible portfolio with identical risk; more explicitly, this means that such preferences can never induce a subject to hold a “loser” in period 1.

### **2.2.2.3 Even Narrower Framing: Individual Stock Accounting**

In order to rationalize a disposition effect that holds on a stock-by-stock basis in the context of our experiment, we need more than prospect-theoretic preferences can provide either alone, or in tandem with a “portfolio accounting” version of narrow framing. Specifically, we need some mechanism by which subjects will care directly about the performance of the stocks that they hold, over and above what preferences over final gains/losses would induce. If we insist on using prospect theory and only prospect theory to generate a stock-by-stock disposition effect, we must assume that subjects frame each of their stock positions as an isolated gamble, and that they have prospect-theoretic preferences over the gains and losses of each stock. Barberis and Huang (2001) refer to this more extreme form of narrow framing as “individual stock accounting.”

By dealing with gains and losses relative to some reference point, in isolation from the reference point itself, prospect theory implicitly introduces something similar to narrow framing. While prospect theory could potentially induce “individual stock accounting,” it will not always do so, nor is it the sole source from which “individual stock accounting” might arise. At least in the context of our experiment, something akin to “individual stock accounting” appears theoretically crucial for generating a disposition effect, but the origin of this “individual stock accounting” (whether prospect theory, or something else) remains unclear from a theoretical perspective.

If we ignore initial purchase decisions, assume that subjects employ individual stock accounting, and assume that subjects have either prospect-theoretic preferences or suitable standard preferences over individual gains/losses, then the usual “increasing curvature” arguments suggest that the subjects will exhibit a disposition effect. However, these assumptions impose some strong restrictions on subjects’ initial purchase decisions. In period 0, subjects have no information about which stocks have which trend, so by symmetry considerations, it

follows that each stock has an expected return of zero, but some positive variance. Therefore, unless they are risk-neutral, subjects should not hold any more of their wealth in stock than they are required to. If a subject chooses to purchase three stocks at date zero rather than purchasing only two stocks and keeping the remainder of her wealth in cash, this will generally reflect a mistake, whether she has standard preferences or prospect-theoretic preferences.

If we find that a subject initially decides to hold more stock than necessary, we would be forced to conclude that either the subject’s decision was determined by factors other than risk and/or loss aversion, or that the subject was risk-neutral. Of course, a subject’s initial decision about which assets to choose only provides local information about their preferences, but evidence of local risk-neutrality could still pose problems for the preference-based explanations discussed earlier. We return to this matter and discuss our related experimental results in section 2.5.2.

### **2.2.3 Regret**

In this subsection, we present a simple formalization and analysis of the “regret” mechanism, as it applies to trading decisions. The regret mechanism (potentially) delivers a stock-specific disposition effect that does not require exogenously imposed “individual stock accounting,” and which is not sensitive to subjects’ levels of risk/loss aversion.

#### **2.2.3.1 Toy Model of Regret**

Rather than positing that subjects derive utility solely from the monetary payoffs that they receive, suppose that subjects derive some hedonic benefit directly from confirming that they took a correct action, and that subjects experience some corresponding detriment from confirming that they took an incorrect action. Although not part of the usual utility framework, this assumption is both intuitively appealing and supported by the psychology literature. Furthermore, an appropriate version of this assumption goes a long way towards delivering a robust, stock-specific disposition effect that does not depend on subjects’ attitudes towards risk. This formulation is similar in spirit to the notion of “realization utility” introduced by

Barberis and Xiong (2009).

Consider the following toy model of the utility of a subject who can either take an action, or not take an action:

$$U = u(c) + (1 - 2\mathbb{I}\{incorrect\})\mathbb{I}\{took\ action\} \quad (2.3)$$

where  $\mathbb{I}\{took\ action\}$  is an indicator function which takes the value 1 if the subject takes the action and 0 otherwise,  $\mathbb{I}\{incorrect\}$  is an indicator function which takes the value 1 if the subject confirms that her action was incorrect, and 0 otherwise, and  $u(\cdot)$  is some utility function defined over some outcome  $c$ . More specifically, suppose that the subject's potential action was to buy a stock, and the "correctness" of this decision is judged in terms of *ex-post* optimality—i.e., the action was incorrect if the subject sells the stock at a price below her purchase price, while the action was correct (or at least not incorrect) otherwise. Also, suppose that the  $u(c)$  term reflects the utility that the subject obtains from her final compensation.

From the perspective of generating a disposition effect, the key detail of the toy model above is that the subject only experiences the disutility directly associated with an incorrect action—which we might think of as "regret"—when she actively sells the stock at a loss. In other words, even if the stock falls in value, the subject does not trigger the regret penalty unless she sells the stock at the depressed value; if the price of the stock recovers before the subject sells it, then the subject does not incur the regret penalty. If the subject did not care about her final compensation at all (i.e., if  $u(c) \equiv 0$ ), then it would be optimal for her to never sell the stock at a loss. If the subject did care about her final compensation, then she might be willing to sell the stock at a loss if doing so would sufficiently increase her expected final compensation, but she would still be more reluctant to sell at a loss than she would be if regret did not factor into her utility.

### 2.2.3.2 The Twin Appeals of "Regret"

Since the "regret" component reduces a subject's propensity to sell a stock at a loss, but leaves unchanged (or even weakly increases) a subject's propensity to sell a stock at a gain, we might

expect that the introduction of regret into our model would result in a disposition effect. If we ignore the initial purchase decision, we may certainly conclude that the introduction of regret induces a disposition effect. However, we already have several mechanisms that do the same thing; we seek a mechanism that not only explains the disposition effect, but which is also compatible with a subject's initial decision to purchase the stock rather than hold cash. From this perspective, it is the *symmetry* of the regret mechanism in (2.3) that comes to our aid. Suppose that at date 0, a subject with preferences given by (2.3) can choose to either take no action and hold cash, or take action by buying a stock whose returns follow a non-degenerate distribution that is symmetric about zero. Assume that if the subject purchases the stock at date 0, then she will definitely sell the stock at date 1, even if this is not the optimal action. Then

$$\begin{aligned}
\mathbb{E}[U|buy\ the\ stock] &= \mathbb{E}[u(c)|buy\ the\ stock] \\
&\quad + \mathbb{E}[1 - 2\mathbb{I}\{incorrect\}|buy\ the\ stock] \\
&= \mathbb{E}[u(c)|buy\ the\ stock] \\
&\quad + \frac{1}{2}(1 - 2) + \frac{1}{2}(1 - 0) \\
&= \mathbb{E}[u(c)|buy\ the\ stock] \\
&\quad \mathbb{E}[U|hold\ cash] = \mathbb{E}[u(c)|hold\ cash] \\
&\quad \mathbb{I}\{took\ action\} \mathbb{E}[(1 - 2\mathbb{I}\{incorrect\})|hold\ cash] \\
&= \mathbb{E}[u(c)|buy\ the\ stock]
\end{aligned} \tag{2.4}$$

This means that the subject's decision about whether or not to buy the stock depends only on her expectation of how the purchase will affect her final compensation—the regret element does not influence her purchase decision.

Note that the calculations above assumed that the subject will definitely sell the stock at date 1, even if this is not the optimal course of action. We are much more interested in the case where the subject chooses the optimal time to sell the stock, since we established that conditional upon owning the stock, subjects who sell in an optimal manner will exhibit

a disposition effect. If we assume that the subject chooses the optimal time to sell the stock, then her expected utility conditional upon buying the stock would weakly increase relative to the value in (2.4):

$$\mathbb{E}[U | \textit{buy the stock}] \geq \mathbb{E}[u(c) | \textit{buy the stock}] \quad (2.5)$$

If the inequality in (2.5) is strict, then we can weaken our various symmetry assumptions somewhat. Rather than assuming as we did in (2.3) that the respective payoffs from confirming correctness and incorrectness are  $+1$  and  $-1$ , we could take them to be some suitable  $\lambda_c$  and  $\lambda_i$ , with  $|\lambda_i| > \lambda_c > 0 > \lambda_i$ . Similarly, we could weaken our assumption that the stock's return process was symmetric about zero. Perhaps more importantly, if the inequality in (2.5) is strict, then it can be optimal for a subject to purchase the stock at date zero even when  $u''(\cdot) < 0$ . In summary, we have now established not only that our “regret” specification generates a disposition effect, but also that the specification is compatible with a subject's initial decision to purchase the stock rather than hold cash.

A final attractive feature of the “regret” approach is that it is natural to suppose that regret is stock-specific. If we consider a subject who makes a collection of decisions,  $\{d_j\}_{j=1}^J$ , we can model this specificity of regret with a utility function of the following form:

$$U = u(c) + \sum_{j=1}^J (1 - 2\mathbb{I}\{d_j \textit{ was incorrect}\}) \quad (2.6)$$

Arguments analogous to those for the single-decision case suggest that preferences of the form (2.6) induce a stock-by-stock disposition effect. This approach of introducing “regret” therefore allows us to generate a stock-by-stock disposition effect without explicitly imposing a separate “individual stock accounting” assumption.

## 2.3 Experimental Design: Details and Implementation

Section 2.2 gave a general overview of our experimental design and sketched some of the theoretical rationale for approaching our research question in that particular manner. This section



describes our experimental design and its implementation in greater detail. The experimental design builds on that Weber and Camerer (1998) in many respects, and more generally, it follows the broad guidelines set forth in Roth (1995).

### **2.3.1 General Setup**

Recall that we define a sequence of 11 periods (numbered 0 to 10) to be a “trading session” and define a collection of 12 such sessions as one run of our experiment. The first session is a “practice session,” during which we assist subjects with any difficulties they may encounter in navigating the experiment; our analysis omits data from these practice sessions, and these practice sessions, as the subjects are informed, do not affect the subject’s performance-based compensation.

Before period 0 of each session, subjects each received an initial endowment of \$120. This initial endowment was divided evenly among exactly three of the following available assets: five simulated risky assets (labeled A-E), and one simulated safe asset (“cash,” labeled F). Subjects in the “choice” treatment groups selected which three assets of the available six they wished to initially hold. Subjects in the “assignment” treatment groups were randomly assigned three of the six available assets to hold initially. Regardless of their treatment group, every subject ended period zero of each session with exactly \$40 invested in each of three distinct assets. The starting prices for all assets are identical and set to \$10.

Once subjects had their initial asset portfolios, they observed the price-change that occurred between periods 0 and 1 for every asset, whereupon the subjects all had the opportunity to trade in the six assets (five risky assets and cash). For the remainder of the session, subjects began each of the next nine periods by observing the price-changes that occurred after the preceding period ended; then, once the most recent price-change had become observable, subjects could trade any or all of the six assets, A-F. Subjects could not sell short any of the risky assets, nor could they hold negative cash positions.

### **2.3.1.1 Checks on Subject Understanding**

The prices of the risky assets were generated by the random process described in detail in section 2.2.1.2. These price processes were explained in simpler terms to the subjects, and the subjects' understanding of the process, as well as their understanding about other basic aspects of the experiment, was tested by means of pre-experiment quizzes. See the chapter appendix for the full set of instructions and quizzes.

After each of sessions 4, 8 and 12, the subjects were asked which of the shares A-E they believed had which trend. In order to create incentives to answer the question correctly, the subjects were informed that they would receive \$1 for a correct set of answers. After the end of the last session, the subjects filled out a post-experimental questionnaire where we collected information on background characteristics such as gender and age, and asked questions regarding which reference point (if any) subjects use, any "rules of thumb" they used when making their trading decisions, and so on. See the chapter appendix for further detail.

### **2.3.1.2 Subject Compensation**

At the end of the experiment, subjects earned 10% of the final value of their portfolio from one randomly chosen session. If subjects were earning (real) money in each session, their perceived wealth/reference point/etc. might change over the course of many sessions, so our payment schedule was chosen so that subjects' [reference points/value functions] or [wealth/risk aversion] would not drift over the course of the experiment. In addition to the performance-based compensation, all subjects received a show-up fee of \$10. The experiment had to be designed so that subjects' incentive pay is always non-negative, and the prohibition on holding negative cash positions ensured that this constraint was satisfied.

## **2.3.2 Implementation Details**

Our experiment was conducted using custom-made computer software. Subjects observed price movements and executed trading and asset-allocation decisions in a fully computerized

setting. See the chapter appendix for screen-shots of the experimental software’s graphic interface. The experiment was implemented at the Harvard Decision Science Laboratory in November 2011 and April - May 2012. We conducted 10 runs in which a total of 155 subjects participated. The subjects were recruited through the SONA system of the laboratory and each subject could participate only once. Participation in the experiment took about 60 minutes and subjects were compensated an average of \$22 (including a show-up fee of \$10).

### **2.3.3 Treatment Groups**

To exogenously introduce the potential to feel regret among a group of subjects, we require those subjects to choose three distinct assets in which to invest the initial endowment that they receive before period 0 of each session. Subjects who do not receive the “choice” treatment are simply assigned three distinct assets at random; these subjects do not choose which assets they initially hold. At the end of period 0, a subject will thus hold \$40 in each of three distinct assets, regardless of which treatment the subject received. This set-up ensures that subjects’ trading behavior during period 1 of a given session can be meaningfully compared across different treatments.

As noted in the introduction, the “choice” treatment requires subjects to engage to a slightly greater extent than does the “assigned initial holdings” alternative treatment. Consequently, the “assigned initial holdings” treatment is not necessarily the ideal control against which to compare the “choice” treatment. To better distinguish the effects attributable to regret from those attributable to increased engagement, we employ a second treatment, “writing,” intended to increase subjects’ engagement without directly changing anything else.

In the default “no writing” case, subjects observe price movements and execute trading and asset-allocation decisions entirely on a computer. In the “writing” treatment, subjects still interface with the computer as they would in the default case, but now subjects also write down on paper which assets they hold in period 0 (regardless of whether the assets were chosen or assigned), and whether each asset’s price increases or decreases between period 0 and period 1. The “writing” treatment doesn’t introduce any new information or directly

affect the act of trading at period 1, but it requires slightly more thought, effort, and time than the default. We partition the “choice” treatment into “choice, writing” and “choice, no writing,” and similarly we partition the “assigned” treatment into “assigned, writing” and “assigned, no writing.”

## 2.4 Experimental Results and Analysis

### 2.4.1 Existence of a Disposition Effect

In all four treatment groups, we find a pronounced disposition effect. Because our experimental design imposes a substantial asymmetry between the expected returns on winners and losers, direct comparison of the proportion of losses realized (“PLR”) to the proportion of gains realized (“PGR”) is not meaningful, since the rational “no disposition effect” benchmark is 1 for PLR, and (with suitable controls, discussed below) 0 for PGR. However, we can meaningfully compare PLR and PGR to their respective benchmarks, and we find that subjects exhibit a much lower propensity to sell losers and a much higher propensity to sell winners, relative to the appropriate benchmarks. Although we nominally find that PLR exceeds PGR, once we account for the artificial asymmetry between winners and losers in our experiment, our results reveal a pronounced and robust disposition effect.

Our experimental design permits a very clean characterization of the disposition effect in terms of PLR. Under very weak assumptions, a subject’s optimal policy would be to sell all her losers in period 1, so a PLR equal to 1 corresponds to no disposition effect, and  $PLR < 1$  represents a disposition effect. Table 2.2 summarizes the experimental data concerning PLR. For every treatment group, the average proportion of losses realized is massively below 1, the no-disposition-effect benchmark. The PLR calculations omit results from sessions where a given subject did not hold any losers at the start of period 1, and from sessions that a given subject did not complete, so the “Obs” column in Table 2.2 presents the total number of subject-session observations included in the calculations.

As discussed in section 2.2.2, the proportion of gains realized (PGR) does not offer an

Table 2.2: Proportion of Losses Realized

		N	Obs	PLR	PLR-1	SE
Choice	No Writing	40	340	0.4866	−0.5134	0.0236
Assigned	No Writing	38	322	0.4894	−0.5106	0.0235
Choice	Writing	40	335	0.5781	−0.4219	0.0235
Assigned	Writing	37	323	0.5075	−0.4925	0.0221
Overall		155	1320	0.5156	−0.4844	0.0116

unambiguous characterization of the disposition effect, essentially because risk aversion might make it optimal for a subject to sell some of her winners, despite the fact that those winners offer a higher expected return than does the riskless asset. However, if a subject is selling winners because she is optimally reducing her exposure to risk, it must be true that she will also sell every loser that she might have been holding. If we restrict attention to the cases in which a given subject did *not* sell every loser that she held at the start of period 1 (and in which she had at least one loser that she could have sold), we can plausibly rule out rational risk-aversion motives and treat  $PGR = 0$  as our “no disposition effect” benchmark. Table 2.3, below, displays the corresponding average proportions of gains realized; in every treatment group, PGR is very significantly larger than the no-disposition-effect benchmark of zero. Analogous to the PLR computations, the PGR estimates exclude results from sessions where a given subject did not hold any winners at the start of period 1, and from sessions that a given subject did not complete. However, the PGR estimates also exclude data from sessions in which a subject sold all of her losers. The “Obs” column in Table 2.3 contains the number of subject-session observations included in the calculations, and the “Obs Excluded” column contains the number of observations excluded because a subject sold all of her losers.

Table 2.3: Proportion of Gains Realized

		N	Obs	Obs Excluded	PGR	SE
Choice	No Writing	40	270	7	0.0951	0.0121
Assigned	No Writing	38	266	11	0.1618	0.0166
Choice	Writing	40	254	0	0.1444	0.0162
Assigned	Writing	37	265	0	0.1112	0.0125
Overall		155	1055	18	0.1279	0.0073

### 2.4.2 Regret and Attention

To analyze the impact of our treatments on the disposition effect, we regress  $PLR$  on treatment indicators, with controls for session-specific fixed effects.<sup>18</sup> Let “*Choice*” denote the indicator variable such that  $Choice = 1$  represents the “choice” treatment, and similarly let “*Write*” denote the indicator variable such that  $Write = 1$  represents the “writing” treatment. Define the variable  $Interact := Choice * Write$ .

Table 2.4, below, displays estimates for the effect of the “choice” treatment, both overall, and partitioned by “writing” treatment. Overall, the average PLR is larger among subjects who received the “choice” treatment than among those who did not, but this difference is at most marginally significant. However, the effect of the “choice” treatment differs markedly between those subjects who received the “writing” treatment, and those who did not. Among subjects in the “no-writing” groups, the “choice” treatment has essentially no effect, while among subjects in the “writing” groups, the “choice” treatment induces a large and highly significant increase in PLR.

---

<sup>18</sup>Recall that during a given session, all subjects observe the same price-paths (up to a relabeling). The effects of both treatments appear stronger if the controls for session-specific fixed effects are omitted, so the results presented are conservative. Analysis of the outcome dependence upon session-specific fixed effects is ongoing.

Table 2.4: Effect of “Choice” Treatment on PLR

	No Writing	Writing	Overall
<i>Choice</i> Coefficient	−0.0047	0.0678	0.0326
SE	0.0332	0.032	0.0231
t-Stat	−0.1419	2.1204	1.4075
N	78	77	155

Estimates for the effect of the “writing” treatment are presented in Table 2.5. As for the “choice” treatment, we present both partitioned and overall estimates. While the “writing” treatment significantly increases PLR on average overall, this increase only manifests to a significant extent in the “choice” treatment groups.

Table 2.5: Effect of “Writing” Treatment on PLR

	Choice	Assigned	Overall
<i>Write</i> Coefficient	0.0932	0.0193	0.0567
SE	0.0331	0.0320	0.0231
t-Stat	3.3869	0.6033	2.4580
N	80	75	155

The results in Tables 2.4 and 2.5 indicate some non-trivial interactions between the “choice” and “writing” treatments, and Table 2.6, below, presents estimates of the joint effects of the two treatments, as well as their interaction. Confirming what the preceding results suggested, we find that the “choice” and “writing” treatments in combination substantially increase PLR, but that neither treatment has a significant independent effect.

Table 2.6: Joint Effects of “Choice” and “Writing” Treatments on PLR

<i>Choice</i> Coefficient	0.0328	−0.0039	—
SE	0.0231	0.0332	—
t-Stat	1.4191	−0.1176	—
<i>Write</i> Coefficient	0.0569	0.0192	—
SE	0.0231	0.0320	—
t-Stat	2.4626	0.6000	—
<i>Interact</i> Coefficient	—	0.0737	0.0841
SE	—	0.0461	0.0268
t-Stat	—	1.5993	3.1341

## 2.5 Discussion

### 2.5.1 Rethinking Regret Aversion

The experimental results do not support the naïve formulation of the “regret aversion” mechanism presented in section 2.2.3.1. Subjects exhibit a stronger tendency to sell losing stocks—i.e., a *reduced* disposition effect—when they chose those stocks, rather than having been assigned them. On its surface, this finding runs counter to the idea that subjects are averse to the regret associated with acknowledging that they made a poor choice, but the statistical significance of these results is not overwhelming. Similarly, the amount of regret that subjects who receive the “choice” treatment potentially face might not be representative of the levels of regret that investors in real markets may experience. Although we cannot draw definitive conclusions about the role that regret aversion may play in generating the disposition effect,



our results present some novel avenues for future study.

The more substantive and unexpected finding from our experiment concerns the powerful influence of small attentional factors on the disposition effect. Although the “writing” treatment requires subjects to expend only slightly more thought, effort, and time than they would in the “no writing” treatment, the “writing” treatment induces a large and highly significant reduction in the disposition effect (as measured by PLR). More interesting still, the effect of this writing treatment is concentrated among subjects who chose their initial asset holdings rather than having their initial holdings assigned. While the full consequences are not yet clear, these findings point to a crucial role for attention and engagement in explaining the disposition effect, and they suggest ways to improve the experimental methodology for investigating the disposition effect going forward.

### 2.5.2 Initial Decision to Hold Cash

Distinct from the results concerning regret aversion and attention, our experiment also helps to illuminate a very different aspect of the disposition effect’s foundations. Because subjects who received the “choice” treatment had the opportunity to choose between holding (riskless) cash and holding a risky asset with the same expected return, these subjects’ initial asset allocation choices shed light on their risk aversion. Similarly, these subjects’ initial portfolio choices potentially reveal whether the subjects are first-order loss averse—in the prospect-theory sense—in a neighborhood of zero.

During period 0 of each session, subjects who received the “choice” treatment selected exactly three assets, in which their endowment that session would initially be invested in equal proportions. The subjects selected among the five risky assets (A-E), and the riskless asset—cash (F). The five risky assets were *ex-ante* identical, and they all had an expected return of zero. Cash also had an expected return of zero, but it differed from the other five assets in that it was riskless. Subjects had to choose 3 of six assets, so if they chose at random, we would expect each asset to be selected 0.5 times per session per subject, on average. However, because cash second-order stochastically dominates the five risky assets,

Table 2.7: Initial Chosen Asset Allocations

	A	B	C	D	E	F
Avg. Times Selected	0.6975	0.6795	0.5981	0.3111	0.2951	0.4187
SE	0.0307	0.0274	0.0366	0.03	0.0261	0.0462

we should expect to see every subject selecting cash at the start of every session.

Table 2.7 summarizes the average number of times that each asset is selected. Several points merit comment. First, among the five risky assets, there is a strong downward trend between A and E, presumably because subjects have a tendency to pick the most readily apparent options. Despite being the last asset listed, though, cash is selected significantly more often than the risky asset (E) listed just before it, suggesting that subjects do distinguish cash from the risky assets. Nevertheless, cash is chosen with significantly less than chance frequency.

Of the 80 subjects who received the “choice” treatment, 44 select cash less often than they would be expected to if they simply picked assets uniformly at random, 42 select cash less than 37% of the time, and 27 never select cash. On the other hand, 14 subjects select cash every time. If subjects were choosing assets uniformly at random, the probability that at least 42 subjects would select cash less than 37% of the time is roughly  $1.9 \times 10^{-6}$ , while the respective probabilities that at least 27 subjects would never select cash, or that at least 14 subjects would always choose cash, are both equal to zero at 32-bit working precision.

Finding that many subjects choose not to initially hold cash is evidence against explanations of the disposition effect that rely on prospect theory *per se*. Whereas a subject with smooth (twice-differentiable) preferences will be approximately risk neutral in any sufficiently small neighborhood, the “kink” in the prospect-theory value function never vanishes in any neighborhood of zero. Since the kink renders second-order stochastically dominated assets undesirable, the usual prospect theory value function cannot perfectly apply to subjects who do not choose to initially hold as much cash as possible. This evidence concerning subjects’

preferences in a neighborhood of zero says nothing about the global properties of those subjects' value/utility functions, specifically about the "S"-shape of the prospect-theory value function.

Interestingly, there is only a negligible difference between the average PLR among subjects who always choose cash initially, and those who never choose cash initially.

## 2.6 Conclusion

To better understand how the disposition effect arises, we ran a laboratory experiment intended to examine if and how "regret aversion"—aversion to admitting mistakes—contributes to generating the disposition effect. Drawing on findings in the psychology literature that a feeling of responsibility is essential for feelings of regret to occur, our basic approach entailed assigning one group of subjects their initial asset allocations, and allowing another group of subjects to choose their initial assets. Only the subjects in the latter group were responsible for deciding their initial asset allocations, and therefore able to regret those decisions.

Although our experimental results resolve few of the questions that we originally sought to answer about the role regret aversion may play in generating the disposition effect, our findings nevertheless contribute to the extant understanding of the disposition effect, while also raising some new questions.

Our results concerning subjects' decisions about whether or not to include cash among the assets that they initially select have several important implications. Beyond posing a moderate new challenge to explanations of the disposition effect based on the typical prospect-theory value function, subjects' observed willingness to choose second-order stochastically dominated assets potentially casts doubt on any preference-based explanation of the disposition effect that relies too heavily on investors' sensitivity to small shifts in relative risk and reward that accompany price movements.

The most important finding from our experiment, however, is the unexpectedly strong influence that attention and engagement have on the disposition effect. Even the minimal increase in engagement/attention induced by the "writing" treatment substantially amelio-

rated the disposition effect by increasing subjects' propensity to sell their losing stocks. More interesting still is the result that both the "writing" treatment and the "choice" treatment had strong and complementary effects when applied together, but neither exerted a significant effect when the other treatment was absent. The effects and interactions of attentional mechanisms offer a promising direction for future research.

## Chapter 3

# Price Targets

### 3.1 Introduction

In this essay, I analyze learning and rational trading dynamics in an asset market model with heterogeneous information and gradual price-adjustment. The fundamental idea that investors heed the informational content of prices when they make trading decisions is thoroughly developed in the large literature on rational-expectations equilibria, as is the related insight that market-clearing prices need not fully reveal all private information when suitable market noise is present.<sup>19</sup> Taking these deep results about equilibrium information-aggregation as a starting point, I introduce a simple form of exogenous information diffusion and derive detailed implications for the trading behavior of a “potentially informed” trader who had previously observed a noisy private signal about an asset’s fundamental value. As information gradually diffuses to the market, the potentially informed trader not only updates his beliefs about the asset’s fundamental value, he also updates his beliefs about the precision of his private signal.

Much of the empirical evidence on individual trading behavior concerns investors’ propensities to sell various assets at a gain versus at a loss, and I structure my analysis to deliver implications with an analogous form.

---

<sup>19</sup>See Grossman (1976), Grossman and Stiglitz (1980), and the related subsequent literature.

### 3.1.1 Realizing Gains and Losses

The tendency of investors to realize their gains and avoid realizing their losses has been documented in a large body of empirical literature. In the seminal paper of this literature, Shefrin and Statman (1985), termed this tendency *the disposition effect*, and they proposed an informal explanation based on the “prospect theory” developed by Kahneman and Tversky (1979). Using trade-level data from a large discount brokerage, Odean (1998) subsequently provided strong empirical evidence for the existence of a disposition effect, and he argued that his results could not be well explained by classical considerations such as portfolio rebalancing, tax-loss selling or belief in mean reversion. Odean’s results did not run counter to Shefrin and Statman’s informal explanation, and as Barberis and Xiong note in their 2009 paper, the disposition effect is now most commonly explained by modeling investor preferences using features borrowed from prospect theory.

Although the claim that prospect-theoretic preferences drive the disposition effect has gained widespread acceptance, empirical evidence casts doubt on its validity. For example, Calvet *et al.* (2009) find that the disposition effect is less pronounced among mutual funds than among individual stocks; while performance chasing may account for part of this reduction, variation in the disposition effect as a function of the identity of the underlying asset is *prima facie* evidence against the common preference-based explanation. Rangelova (2000) presents much stronger evidence in this vein. Using Odean’s (1998) data, Rangelova finds that the disposition effect is concentrated in large-cap stocks, and that the disposition effect actually reverses for stocks in the bottom quintile of market cap. Moreover, Rangelova shows that her results are robust to individual effects, margin calls, and a variety of other controls. Given this variation in trading behavior, I will use the term *disposition behavior* to refer to where trading tendencies fall on the disposition effect/anti-disposition effect spectrum; I will reserve the term *disposition effect* to specifically refer to those instances where investors realize gains more readily than losses. The prospect-theoretic preference explanation of the disposition effect has also proven tenuous from a theoretical standpoint. Barberis and Xiong (2009) demonstrate that popular preference-based models sometimes fail to deliver a disposition

effect, and the failures do not bear any obvious relationship to the empirically observed settings in which the disposition effect reverses or is not observed.

### 3.1.2 Learning, Rational Expectations, and Standard Preferences

Ranguelova suggests that her results could be explained by a model in which investors had “price targets,” which they updated at rates that depended on the characteristics of the underlying stock. In this essay, I formalize this intuition and model the trading behavior of a rational investor who sets a price target and learns from price movements. I consider a single investor—assumed small relative to the aggregate market—who observes a private “price-target” signal about the fundamental value of a stock; this signal may either reveal the true fundamental value of the stock, or it may contain no information whatsoever about fundamental value. I will refer to the investor who receives this signal as the “potentially informed trader,” or “PIT.” Intuitively, the PIT’s signal could either be true private information that has not yet been incorporated into the market price, or it could merely be stale information with idiosyncratic noise. Information about the true fundamental value of the stock diffuses to other investors according to some known stochastic process, and the arrival of this information affects the market price, so the PIT uses the evolution of the price to update his beliefs about whether his signal was informative. The binary informative/non-informative signal structure emphasizes the novel aspect of this model, namely that the PIT uses the behavior of the market price to learn about whether or not he has private information.<sup>20</sup>

Section 3.2 develops a simple model of trading and belief-updating in the “price-target” framework. The PIT’s optimal updating scheme will depend on various underlying characteristics of the stock, and that updating scheme will determine how the stock’s price movements affect the PIT’s trading decisions. Section 3.3 discusses how this price-target framework helps to explain a number of existing empirical findings, and it presents several new empirical predictions. The question of whether many of these empirical predictions are actually

---

<sup>20</sup>Previous research has analyzed the issue of learning about the precision of private signals received by *other investors*; see Gervais (1996). By contrast, the present analysis concerns an investor learning about the precision of *his own* private signal.

testable is non-trivial, and section 3.4 presents theoretical results demonstrating that mild regularity conditions ensure that the model's predictions can indeed be tested. Section 3.5 examines how time can be explicitly introduced into the price-target model, discusses relaxing the binary informative/non-informative signal structure, and addresses the relationship between the price-target model and prospect-theoretic preferences. Section 3.6 concludes. Full mathematical derivations are presented in the Chapter 3 Appendix.

## 3.2 A Rational Model With Price Targets

### 3.2.1 Market Structure

There are two assets, a risk-free asset, with a return normalized to zero, available in perfectly elastic supply (“cash”), and a risky asset (“stock”), available in net supply normalized to unity. The fundamental value of the stock is  $V$ , with  $V \sim N(\mu, \sigma_V^2)$ , and the value of  $V$  is drawn before the model begins. Time is discrete, consisting of periods  $\{0, 1, 2\}$ , and  $V$  is publicly revealed in period 2.

The market consists of a large number,  $N$ , of homogeneous investors, each with mean-variance utility over the fundamental value of next-period wealth, given by

$$U(W_{t+1}) = \mathbb{E}_t[W_{t+1}] - \frac{\gamma}{2} \text{Var}_t[W_{t+1}]$$

Investors myopically optimize their portfolios with respect to this utility function in each period. At times  $t \in \{0, 1\}$ , investors believe  $V \sim N(\mathbb{E}_t^M[V], \text{Var}_t[V])$ , where  $\mathbb{E}_t^M[V]$  and  $\text{Var}_t[V]$  are the expectation and variance, respectively, of fundamental value, conditional upon public information available at time  $t$ . Assume that  $\mathbb{E}_0^M[V] = \mu$ .

At each date  $t \in \{0, 1\}$ , investors solve for the optimal amount to invest in the stock,  $x_t^*$ :

$$\begin{aligned} & \max_x \left\{ \mathbb{E}_t[W_{t+1}] - \frac{\gamma}{2} \text{Var}_t[W_{t+1}] \right\} \\ \implies & x_t^* = \frac{\mathbb{E}_t^M[V] - P_t}{\gamma \text{Var}_t[V]} \end{aligned}$$



Because net supply is normalized to unity, the market-clearing condition for the stock is

$$P_t = Nx_t^*$$

It follows that

$$\begin{aligned} P_t^* &= \frac{N\mathbb{E}_t^M[V]}{N + \gamma \text{Var}_t[V]} \\ x_t^* &= \frac{\mathbb{E}_t^M[V]}{N + \gamma \text{Var}_t[V]} \end{aligned}$$

Now, suppose that there are some noise traders active in each period  $t$ , and they demand (or supply) some random quantity  $\delta_t$  inelastically, so that residual supply is  $1 - \delta_t$ , and the market price  $\tilde{P}_t$  is given by  $\tilde{P}_t = P_t/(1 - \delta_t)$ . Assume that  $\delta \sim U[-c, c]$  for some small positive constant  $c$ , so that the market price will stay in some small  $c$ -neighborhood of  $\frac{N\mathbb{E}_t^M[V]}{N + \gamma \text{Var}_t[V]}$ . At this stage, the noise traders are introduced simply to circumvent no-trade results.

### 3.2.2 Information Structure

At time 0, the fundamental value of the stock,  $V$ , is not known to investors, and I assume that  $\frac{N\mu}{N + \gamma\sigma_V^2} \equiv P_0 \neq V$ . For concreteness, I will suppose that  $P_0 < V$ , as this falls naturally out of the risk-compensation framework, but there would be little conceptual difference in supposing  $P_0 > V$ .

#### 3.2.2.1 Public Information

At time  $t = 1$ , a noisy signal of  $V$ , call it  $\Psi$ , is publicly revealed. Assume that  $\Psi = V + \epsilon_1$ , where  $\epsilon_1 \sim N(0, \sigma_\epsilon^2)$  and  $\epsilon_1$  is independent of  $V$ . Then, in period 2,  $V$  is publicly revealed, so market expectations change, all uncertainty is resolved,  $\mathbb{E}_2^M[V] = V = P_2$ , and  $\text{Var}_2[V] = 0$ .

#### 3.2.2.2 Private Information

Consider a single investor, to whom I will refer as the *potentially informed trader*, or *PIT*. At date 0, the PIT receives a signal,  $S$ , which he alone observes. With probability  $\tau < 1$ ,  $V = S$ ,

i.e., the signal reveals the fundamental value of the stock. However, with probability  $1 - \tau$ , the signal is completely uninformative, and has no information about the stock's fundamental value. I will call a signal *true* if it reveals the fundamental value of the stock, and *false* if it is instead uninformative. This binary true/false signal structure simplifies the model and leads to more tractable analysis, but other specifications for the PIT's signal deliver the same qualitative results. See section 3.5.2 for further discussion.

**The PIT's Optimal Allocation** Consider the portfolio optimization problem facing the PIT. At date  $t$ , he solves

$$\begin{aligned} & \max_x \left\{ \tau_t \mathbb{E}[(V - P_t)x | V = S] + (1 - \tau_t) \left( (\mathbb{E}_t^M[V - P_t]x) - \frac{\gamma}{2} \text{Var}_t[V - P_t]x^2 \right) \right\} \\ \Rightarrow \quad x_t^\dagger &= \frac{\tau_t (S - \mathbb{E}_t^M[V]) + (\mathbb{E}_t^M[V] - P_t)}{(1 - \tau_t)\gamma \text{Var}_t[V]} \end{aligned}$$

We can express the PIT's optimal allocation to the stock,  $x_t^\dagger$  in terms of the average investor's optimal allocation,  $x^*$ :

$$\begin{aligned} x_t^\dagger &= \frac{\tau_t}{(1 - \tau_t)} \frac{S - P_t}{\gamma \text{Var}_t[V]} + x_t^* \\ &= \frac{\tau_t (S - \mathbb{E}_t^M[V])}{(1 - \tau_t)\gamma \text{Var}_t[V]} + \frac{1}{1 - \tau_t} x_t^* \end{aligned}$$

This expression has an intuitive interpretation: the PIT's demand for the stock is increased relative to that of other investors both by the potential excess return (the  $\frac{\tau_t (S - \mathbb{E}_t^M[V])}{(1 - \tau_t)\gamma \text{Var}_t[V]}$  term), and by the implicit reduction in the riskiness of the stock (the  $\frac{1}{1 - \tau_t}$  coefficient on  $x_t^*$ ). It is also useful to have an explicit expression for the PIT's excess demand, relative to that of an average investor, so define

$$\begin{aligned}
\Delta_t &\equiv x_t^\dagger - x_t^* \\
&= \frac{\tau_t (S - \mathbb{E}_t^M[V])}{(1 - \tau_t)\gamma \text{Var}_t[V]} + \frac{\tau_t}{1 - \tau_t} x_t^* \\
&= \frac{\tau_t}{(1 - \tau_t)} \frac{S - P_t}{\gamma \text{Var}_t[V]}
\end{aligned}$$

### 3.2.3 Investors' Belief Updating

Let  $\psi$  denote the realized value of  $\Psi$ ; following the revelation of  $\psi$ , non-PIT investors update their priors and assign  $V$  a posterior (Gaussian) distribution with the following moments:

$$\begin{aligned}
\mathbb{E}^M[V|\Psi = \psi] &= \frac{\sigma_\epsilon^2 \mu + \sigma_V^2 \psi}{\sigma_\epsilon^2 + \sigma_V^2} \\
\text{Var}^M[V|\Psi = \psi] &= \left( \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_V^2} \right) \sigma_V^2
\end{aligned}$$

The updating problem facing the PIT is clearly different, since his prior beliefs about  $V$  differ from those of the other investors. If the PIT's price-target signal is true, then  $\Psi \sim N(S, \sigma_\epsilon^2)$ , while if it is false,  $\Psi \sim N(\mu, \sigma_V^2 + \sigma_\epsilon^2)$ , so we have two different conditional prior densities for  $\Psi = \psi$ , call them  $g(\psi|true)$  and  $g(\psi|false)$ . The PIT uses the signal  $\psi$  to update his estimate of the probability  $\tau$  that his private price-target signal is true.

Define the likelihood ratio

$$\begin{aligned}
\Lambda_\psi &\equiv \frac{g(\psi|true)}{g(\psi|false)} \\
&= \sqrt{1 + \frac{\sigma_V^2}{\sigma_\epsilon^2}} \exp \left( \frac{\sigma_\epsilon^2 (\psi - \mu)^2 - (\sigma_\epsilon^2 + \sigma_V^2) (\psi - S)^2}{2\sigma_\epsilon^2 (\sigma_\epsilon^2 + \sigma_V^2)} \right)
\end{aligned}$$

Using Bayes' rule, the PIT updates his estimate of  $\tau$ :

$$\tau_1 = \frac{\Lambda_\psi \tau_0}{\Lambda_\psi \tau_0 + (1 - \tau_0)}$$

Note that  $\tau_1 > \tau_0 \iff \Lambda_\psi > 1$ .

### 3.2.4 Investor Trading Following Revelation of $\Psi$

After observing  $\psi$  and updating her beliefs, each non-PIT investor has a demand for the stock given by

$$\begin{aligned} x_1^* &= \frac{\mathbb{E}_1^M[V] - P_1}{\gamma \text{Var}_1[V]} \\ &= \frac{\sigma_\epsilon^2 \mu + \sigma_V^2 \psi}{\gamma \sigma_\epsilon^2 \sigma_V^2} - \frac{\pi_\epsilon + \pi_V}{\gamma} P_1 \end{aligned}$$

where  $\pi_\epsilon \equiv 1/\sigma_\epsilon^2$  and  $\pi_V \equiv 1/\sigma_V^2$  are precisions.

By market-clearing (assuming that the excess demand by the PIT is negligible, and ignoring noise traders), we have

$$\begin{aligned} P_1 &= N x_1^* \\ &= \frac{N (\sigma_\epsilon^2 \mu + \sigma_V^2 \psi)}{N (\sigma_\epsilon^2 + \sigma_V^2) + \gamma \sigma_\epsilon^2 \sigma_V^2} \\ \implies x_1^* &= \frac{\sigma_\epsilon^2 \mu + \sigma_V^2 \psi}{N (\sigma_\epsilon^2 + \sigma_V^2) + \gamma \sigma_\epsilon^2 \sigma_V^2} \end{aligned}$$

However, because the supply of the stock is fixed and the non-PIT investors are assumed to be identical, it follows from the symmetry of the situation (again assuming that the excess demand by the PIT is negligible, and ignoring the effects noise traders) that

$$\frac{x_0^*}{P_0} = \frac{x_1^*}{P_1} = \frac{1}{N}$$

For the PIT (whose demand, we assume, has a negligible impact on market price), new optimal demand for the stock is given by

$$x_1^\dagger = (\pi_\epsilon + \pi_V) \frac{\tau_1 (S - P_1) + (1 - \tau_1) (\mathbb{E}_1^M[V] - P_1)}{(1 - \tau_1) \gamma}$$

The number of shares of the stock that the PIT demands at  $t = 1$  is given by

$$\frac{x_1^\dagger}{P_1} = \frac{1}{N} + \frac{\pi_\epsilon + \pi_V}{\gamma} \frac{\tau_1}{1 - \tau_1} \frac{S - P_1}{P_1}$$

Similarly, the number of shares of the stock that the PIT demands at  $t = 0$  is given by

$$\begin{aligned} \frac{x_0^\dagger}{P_0} &= \frac{\tau_0(S - P_0) + (1 - \tau_0)(\mathbb{E}_0^M[V] - P_0)}{P_0(1 - \tau_0)\gamma\sigma_V^2} \\ &= \frac{1}{N} + \frac{\pi_V}{\gamma} \frac{\tau_0}{1 - \tau_0} \frac{S - P_0}{P_0} \end{aligned}$$

Since each non-PIT holds the same number of shares,  $\frac{1}{N}$ , in every period, the PIT's excess demand (in terms of shares) relative to that of a non-PIT can be meaningfully compared between periods. From the calculations above, we find

$$\frac{\Delta_1/P_1}{\Delta_0/P_0} = \frac{\pi_\epsilon + \pi_V}{\pi_V} \frac{P_0}{P_1} \left( \frac{S - P_1}{S - P_0} \right) \Lambda_\psi$$

This expression gives very clean insights into the PIT's trading behavior.<sup>21</sup> When price decreases, the potential reward if the PIT's signal is true is relatively larger, so *ceteris paribus* this increases the number of shares that the PIT optimally wishes to hold. However, the public signal will also cause the PIT to update his estimate of the probability  $\tau$  that his private signal was true. If the public signal provides evidence that the PIT's private signal was false (which manifests as the likelihood ratio term,  $\Lambda_\psi$ , in the expression for  $\frac{\Delta_1/P_1}{\Delta_0/P_0}$ ), then this tends to decrease the number of shares that the PIT wishes to hold. The former effect weakens relative to the latter as the precision of the public signal increases relative to that of the PIT's private signal.

---

<sup>21</sup>This expression can be further expanded using the definition of the likelihood ratio:  $\frac{\Delta_1/P_1}{\Delta_0/P_0} = \sqrt{\frac{\sigma_\epsilon^2 + \sigma_V^2}{\sigma_\epsilon^2}} \exp\left(\frac{\sigma_\epsilon^2(\psi - \mu)^2 - (\sigma_\epsilon^2 + \sigma_V^2)(\psi - S)^2}{2\sigma_\epsilon^2(\sigma_\epsilon^2 + \sigma_V^2)}\right) \frac{\pi_\epsilon + \pi_V}{\pi_V} \frac{P_0}{P_1} \left(\frac{S - P_1}{S - P_0}\right)$

### 3.2.5 Price Movements as the Basis for the PIT's Learning

Rather than assuming that  $V$  itself is publicly revealed in period 2 and the model ends, suppose now that in each period  $t = 2, 3, 4, \dots$ , a new noisy signal of  $V$ ,  $\Psi_t$ , is publicly revealed. For each  $t$ ,  $\Psi_t = V + \epsilon_t$ , where  $\epsilon_t \sim i.i.d. N(0, \sigma_\epsilon^2)$ , and each  $\epsilon_t$  is independent of both  $\epsilon_1$  and  $V$ . Since the error terms in each public signal are independent and have uniformly bounded variance,  $\mathbb{E}_t^M [V | \mathcal{F}_t] \rightarrow V$  *a.s.*. All of the results from subsections 3.2.3 and 3.2.4 extend immediately to each new period of this multiple-public-signal setting.<sup>22</sup>

The implausible assumption that every market participant literally observes a sequence of noisy signals of the stock's fundamental value can readily be replaced with something weaker and more realistic. If some adequate subset of investors sees a noisy signal directly, the subsequent innovation in the market price will partially reveal what those investors saw and provide the PIT with some amount of new information.

As the simplest case, suppose that all investors except for the PIT see the first “public” signal  $\psi$ . The new market-price (ignoring noise-trader effects) is

$$P_1 = \frac{N(\sigma_\epsilon^2 \mu + \sigma_V^2 \psi)}{N(\sigma_\epsilon^2 + \sigma_V^2) + \gamma \sigma_\epsilon^2 \sigma_V^2}$$

Routine algebra shows that  $P_1$ , the market price ex-noise, is a sufficient statistic for  $\psi$ :

$$\psi = \frac{\sigma_\epsilon^2}{\sigma_V^2} (P_1 - \mu) + \frac{N + \gamma \sigma_\epsilon^2}{N} P_1$$

However, due to noise trader activity, the PIT observes only a noisy signal of  $P_1$ , so he faces the additional signal-extraction problem of inferring the value of  $\psi$  from the market price. This means that the informational content of price innovations in the market for the stock now plays a role in how the PIT updates his beliefs about  $\tau$ . The more information that a change in the stock's price reveals, the greater the influence on the PIT's updated beliefs about  $\tau$ .

---

<sup>22</sup>Non-PITs assign the prior  $\Psi_t \sim N(\mathbb{E}_{t-1}^M [V], \text{Var}_{t-1} [V] + \sigma_\epsilon^2)$ ; when conditioning on his signal being false, the PIT also assigns  $\Psi_t$  this prior.

### 3.2.5.1 Multiple PITs

Up to this point, I have assumed that a single investor, the PIT, received a private “price-target” signal. However, without loss of generality, we could suppose that the PIT actually represents a collection of many small investors. Provided that the PIT (collection) remains small enough relative to the overall market and the level of noise trader activity that the PIT’s trades have a negligible impact on price, the preceding analysis holds unchanged. If the PIT *did* have a non-negligible impact on price, then the PIT’s inference from prices would involve the additional task of filtering out the effects that reflected only the information from his original signal.

## 3.3 Empirical Implications

The price-target model developed in the preceding section has considerable empirical content, and this section presents and discusses some of the model’s most important implications.

### 3.3.1 Different Disposition Behavior for Different Assets

The central point of subsection 3.2.5 is that because the PIT uses price movements to update his beliefs about whether his signal was true, the PIT’s beliefs will change more when prices reveal more about other investors’ private information. Similarly, following the remarks at the end of subsection 3.2.4, the PIT will change his beliefs more when the precision of other investors’ private information is larger relative to the precision of his signal. Both the informativeness of prices, and the scope for other investors to have more accurate information than you, systematically vary somewhat across different assets.

These results about the sensitivity of the PIT’s beliefs to price innovations readily translate into results about the PIT’s disposition behavior. First, suppose that the PIT does not update his beliefs about  $\tau$  at all. The stock becomes unambiguously more attractive when the price decreases, and the stock becomes unambiguously less attractive when the price increases, so the PIT will only ever sell at a gain. Next, suppose that the PIT drastically revises his

beliefs about  $\tau$  on the basis of price movements. Then when the price decreases, the massive increase in the probability that the PIT's signal was false will overwhelm the potential increase in profits if his signal were true, and a symmetric situation occurs when the price increases, so the PIT will frequently sell at a loss, but more rarely sell at a gain. When price movements have little effect on the PIT's beliefs about  $\tau$ , the PIT will tend to exhibit a disposition effect, while he will tend to exhibit an anti-disposition effect when price movements exert a strong influence on the PIT's beliefs about  $\tau$ . Hence the price-target model links the PIT's disposition behavior to the underlying asset's market characteristics.

In more explicit terms, the price-target model implies that the PIT will exhibit a standard disposition effect among stocks whose price innovations are *unlikely* to convey information about fundamental value that the PIT did not already know, but that the PIT will exhibit an anti-disposition effect among stocks whose price innovations are *likely* to convey information about fundamental value that the PIT did not already know. Since the extent to which price movements convey private information is typically greatest for small-cap stocks and smallest for large-cap stocks, the price-target model can explain the systematic differences in disposition behavior as a function of market cap observed by Rangelova (2000).

Although the price-target model was designed to generate disposition behavior that depended on the characteristics of a given asset, the form of that dependence was not exogenously imposed, so the fact that the model offers an explanation for Rangelova's results is non-trivial. Moreover, the price-target model delivers novel empirical predictions for the covariation between disposition behavior and asset characteristics *other than market cap* that provide some measure of the informativeness of price movements. For example, the model implies that disposition behavior should vary with characteristics such as analyst coverage and bid-ask spread.

Finally, the price-target model also has at least one interesting implication that does not depend on the updating/learning mechanism. The model suggests that investors should exhibit no disposition effect—or anti-disposition effect—for mutual funds, since there is no meaningful notion of a price-target for a mutual fund. Even without the rational learning



structure, the price-target model can help to explain the empirical finding that disposition behavior differs between stocks and mutual funds.

### 3.3.2 The Precision of Private Signals

At a very general level, the price-target model implies that disposition behavior will be affected in a specific and well-defined manner by the relative reliability/precision of public vs. private information. Variation across different assets of this relative reliability/precision forms the basis for the predictions discussed in subsection 3.3.1, but the price-target model's implications are broader.

Intuitively, if we can identify stocks about which we expect a particular investor to have unusually reliable private information, then we should see an increased disposition effect by that investor for those stocks, relative to his other investments (holding constant asset-specific characteristics). The notion of “more reliable private information” does not correspond to the quantity  $\tau$  in the model, but rather to something more subtle. In the language of section 3.2.5, assume that an investor holds several different stocks, and assume that the public-information estimate of  $\sigma_\epsilon^2$  is the same for all of those stocks. The investor has “more reliable private information” about a certain stock if he knows that the true value of  $\sigma_\epsilon^2$  for that stock is larger than the public-information estimate. In other words, the idea here is not necessarily that the investor is likely to know fundamental value, but rather that other people are less likely to learn anything that the investor doesn't already know. The investor's private information is unusually “reliable” in the sense that it is unusually comprehensive.

In general, the reliability of a given investor's private information about particular stocks is not something that can be easily measured, but among mutual fund managers, we can use some measure of “proximity” as an instrument for more reliable private information. It is well documented that mutual fund managers tend to make abnormal returns on stocks of firms that are close to them physically, or close to them in a “social network of managers” sense, (cf. Hong, Kubik and Stein (2005) and Cohen, Frazzini and Malloy (2008)) so the extant literature contains several suitable measures of proximity.

### 3.4 Foundations for Empirical Tests

From the perspective of testing many predictions of the price-target model, two related questions about the measurement of disposition behavior are of paramount importance:

1. If we observed the trades of all participants in the market for a given stock, could a disposition (or anti-disposition) effect be observed in aggregate, or is “disposition” conserved on a market-wide basis, like net trading profit?
2. Does the presence of (anti-) disposition effects among the PIT population for a given stock induce a corresponding effect on the average (anti-) disposition effect measured for that stock?

Essentially, the price-target model delivers clean predictions about the disposition behavior of PITs, but because we cannot typically observe who the PITs are, empirical assessment is feasible only if these predictions about the aggregate behavior of all investors or the behavior of the average investor.

In the remainder of this section, I demonstrate that under mild regularity conditions, 1) disposition behavior can be aggregated over an entire market to provide a meaningful statistic, and 2) the disposition behavior of the PIT population will generally induce corresponding aggregate disposition behavior.<sup>23</sup>

#### 3.4.1 Characterizing Disposition Behavior in a Theoretical Setting

The standard method for measuring the average disposition effect that a given investor exhibits entails taking a “snapshot” of that investor’s portfolio each time that he sells stock, and marking this as a “realization” event. At the date of a realization, the stocks in the investor’s portfolio are characterized as either “winners” or “losers,” depending on whether they are above or below their purchase price, respectively, as are the stocks that the investor bought

---

<sup>23</sup>The issues discussed in this section are somewhat deep, and they are relevant to a non-trivial body of behavioral finance literature beyond this essay. Although I have limited my exposition to the simplest cases, generalizations of the issues in this section suggest a number of purely econometric questions which may be of independent interest.

or sold; these data are then used to calculate the “proportion of gains realized” (PGR) and the “proportion of losses realized” (PLR). Similarly, by aggregating these four measures both over time and across investors for a single stock, following Rangelova (2000), this method can be adapted to measure the average disposition effect among investors in that stock.

In empirical applications, an important obstacle is presented by the fact that investors may face unobserved factors that influence the times at which they choose to sell stock, and those unobserved factors could be correlated with price movements. Recording measurements at the dates upon which realizations occur helps to deal with this problem, so the technique is important for empirical work. However, this “realization-dated sampling” obscures the theoretical relationships between measurements of disposition effects and underlying investor trading behavior. For theoretical analyses, the cumbersome machinery of realization-dated sampling is unnecessary, and in its place we can simply sample every period. The PLR and PGR that result from every-period sampling (call these PLRE and PGRE, respectively) will clearly be smaller in absolute magnitude than their respective analogs calculated via realization-dated sampling, but the ordering will be preserved; i.e. if  $\mathbb{E}[PLR] > \mathbb{E}[PGR]$  then  $PLRE > PGRE$ , and if  $\mathbb{E}[PLR] < \mathbb{E}[PGR]$ , then  $PLRE < PGRE$ .

The quantities PLRE and PGRE are always defined with respect to the same sequence of sampling dates (i.e., every period), so their expectations can be computed directly from the trading rule and price-process for a given stock. By contrast, computing the expectations of the traditional PLR and PGR requires knowledge of the trading rules and price-processes of every stock in the investor’s portfolio. As a result, if different investors hold different portfolios, PLRE and PGRE naturally lend themselves to cross-stock comparisons, whereas expectations of PLR and PGR do not.

In short, PLRE and PGRE serve as the analytical tools necessary to address the questions raised at the beginning of this section.

### 3.4.2 Can an Aggregate Disposition Effect Exist?

To illustrate that (and how) an aggregate disposition effect can exist, consider a stock with a single, indivisible share, and assume that the stock's log-price,  $p$ , follows a simple, symmetric random walk on the integers. Consider a single investor,  $d_0$ , who purchases that stock at date  $t = 0$  at a log-price normalized to zero. Let  $\tau_g$  denote the first date at which  $p = 2$ , and let  $\tau_l$  denote the first time at which  $p = -3$  ("g" and "l" for "gain" and "loss," respectively).

The investor  $d_0$  trades according to the following strategy: sell at time  $\hat{t}$ , where  $\hat{t} \equiv \min\{\tau_g, \tau_l, 4\}$ . Since  $\hat{t}$  is a stopping time and the log-price process is a martingale, the log-price process stopped at  $\hat{t}$  is a martingale, so the investor's expected sale price is equal to his purchase price—on average, the investor breaks even by following this trading rule. Intuitively, in the first 3 periods after purchase, the investor sells the stock either when its log-price hits an upper target, or when its log-price falls so low that the investor decides that his initial assessment was mistaken; in period 4, the investor no longer thinks that he has any meaningful information about future price movements, so he sells the stock at the current market price, if he has not done so already. Note that this trading rule is qualitatively similar to a strategy that a PIT might follow. Using the specified trading strategy, we can calculate the number of total gains (sampling every period), total losses (sampling every period), realized gains, and realized losses along each possible sample path of the stopped log-price process.

When  $d_0$  sells the stock, a different investor, call him  $d_1$ , must purchase it. Gains and losses for  $d_1$  are determined relative to the log-price at which he buys the stock from  $d_0$ , so without loss of generality, we can normalize this initial log-price to zero. Clearly, without loss of generality, we can also re-normalize the date so that the period in which  $d_1$  initially buys the stock is labeled as zero. Now, suppose that  $d_1$  follows the same trading strategy as did  $d_0$ , that is, he sells at the (re-normalized) time  $\hat{t}$ . Then *the setting for  $d_1$  is identical to the setting for  $d_0$* . If we continue in this manner for a sequence of investors who all follow the same trading strategy,  $\{d_j\}_{j=1}^\infty$ , with  $d_j$  selling the stock to  $d_{j+1}$ , we will get the sequence  $\{(\# \text{total losses}, \# \text{total gains}, \# \text{realized losses}, \# \text{realized gains})_j\}_{j=0}^\infty$  that captures the trading behavior of every investor who participated in the market, and the sample average of each

component (e.g. # total losses) converges to the expectation of that component for investor  $d_0$ .<sup>24</sup> We know the stochastic process for the log-price process, so we can assign probabilities to each possible sample path that  $d_0$  could face and directly calculate the expectation for each of these components. With the given example parameter values, and using the convention that neither a loss nor a gain is recorded if sale price equals purchase price, these expectations are

$$\begin{aligned}\mathbb{E} [\text{\#total losses}] &= 23 \\ \mathbb{E} [\text{\#total gains}] &= 18 \\ \mathbb{E} [\text{\#realized losses}] &= 5 \\ \mathbb{E} [\text{\#realized gains}] &= 6\end{aligned}$$

and thus

$$\begin{aligned}\frac{\mathbb{E} [\text{\#realized losses}]}{\mathbb{E} [\text{\#total losses}]} &\approx 0.217 \\ \frac{\mathbb{E} [\text{\#realized gains}]}{\mathbb{E} [\text{\#total gains}]} &\approx 0.333\end{aligned}$$

Of course, by Jensen's inequality,  $\frac{\mathbb{E} [\text{\#realized losses}]}{\mathbb{E} [\text{\#total losses}]} \neq \mathbb{E} \left[ \frac{\text{\#realized losses}}{\text{\#total losses}} \right]$ , so we might instead wish to directly calculate (e.g.)  $\mathbb{E} \left[ \frac{\text{\#realized losses}}{\text{\#total losses}} \right]$  for  $d_0$ , noting that

$$p \lim_{J \rightarrow \infty} \frac{1}{J} \sum_{j=0}^J \frac{(\text{\#realized losses})_j}{(\text{\#total losses})_j} = \mathbb{E} \left[ \frac{\text{\#realized losses}}{\text{\#total losses}} \right]$$

The only difficulty with this approach is that the sum on the left-hand side may include terms of the form “zero/zero,” so some convention must be adopted to deal with these. In the case

---

<sup>24</sup>By introducing this sequence of investors, we can also justify our assumptions about the price process. Suppose that at each date, there are two identical investors in the market in addition to the one who owns the stock, and these other investor offers to buy the stock at some price (they compete via Bertrand competition, so their offers are driven to exactly their willingness to pay; the stock is allocated to one of them at random). If we define this offer to be the market price in that period, then we can generate any price process that we desire through an appropriate choice of stochastic process for the beliefs that these other investors hold about the value of the stock.

at hand, if  $\frac{0}{0} \equiv 1$ , then

$$\begin{aligned}\mathbb{E} \left[ \frac{\# \text{realized losses}}{\# \text{total losses}} \right] &= 0.484 \\ \mathbb{E} \left[ \frac{\# \text{realized gains}}{\# \text{total gains}} \right] &= 0.552\end{aligned}$$

while if the  $\frac{0}{0}$  terms are omitted, the results are

$$\begin{aligned}\mathbb{E} \left[ \frac{\# \text{realized losses}}{\# \text{total losses}} \right] &= 0.175 \\ \mathbb{E} \left[ \frac{\# \text{realized gains}}{\# \text{total gains}} \right] &= 0.283\end{aligned}$$

These results suggest that the details of the averaging scheme may be important for empirical work, but they also demonstrate that an aggregate disposition effect can exist. Moreover, it follows from the symmetry of this example that an aggregate anti-disposition effect could also exist. Because the reference point for gains and losses resets when the stock changes hands, disposition behavior need not “wash out” on average, so average/aggregate disposition behavior is a meaningful quantity. This is also true because PLRE and PGRE are price-path-dependent, so these measures of disposition behavior are not conserved in the mathematical sense. Finally, although the preceding analysis employed an infinite sequence of investors, note that it suffices to have just two investors whose beliefs about the value of the stock may differ and vary over time.

### 3.4.3 Will a Disposition Effect Show Up for the Average Investor, or Only for PITs?

The analysis in the preceding subsection can be extended to characterize the behavior of the measured “average” disposition effect when investors do not all follow the same trading strategy.

Consider the example from subsection 3.4.2, but now suppose that on each odd-numbered transaction, an  $d$ -type investor, say  $d_{2j}$ , sells the stock to a new type of investor, a “ $q$ -type”

investor, say  $q_{2j+1}$ . All of the  $d$ -type investors follow the trading strategy discussed earlier; they sell the stock at the stopping time  $\hat{t}$ . The  $q$ -type investors, however, follow a different strategy; they hold the stock for  $h = 4$  periods, and then sell it, regardless of the market price. The  $q$ -type investors can be thought of as passive investors<sup>25</sup>. Although this setting seems different from that of the first example, the market still has a repeated structure. Thus if we consider blocks of market observations, each of which covers one  $d$ -investor and one  $q$ -investor, analysis proceeds exactly as it did in the first example.

By a symmetry argument, it follows that in expectation,  $PLRE = PGRE$  among  $q$ -type investors. Consequently, the difference  $PLRE - PGRE$  will be closer to zero when measured over the entire population of investors than it would be if measured only over  $d$ -type investors, but the expected sign of this difference will not change as a result of introducing the  $q$ -investors. By varying the ratio of  $q$ -investors to  $d$ -investors (and noting that their order in the trading sequence doesn't matter, as long as their relative distribution preserves the ergodicity of the desired sample averages), and by varying the holding period  $h$  of the  $q$ -investors, the relative influence of the passive investors can be made arbitrarily large or small, and the same conclusions hold. Thus the empirical predictions of the model in this essay should be observable in average measures of disposition behavior, even though the model only predicts effects among the PIT population.

## 3.5 Discussion

### 3.5.1 Price Targets and the Passage of Time

The price-target model of section 3.2 introduces a rudimentary structure in which to analyze the effects of the rate at which information about fundamental value becomes public. This rate depends in part upon the extent to which price movements reveal new private information, but it also depends upon the rate at which new private information arrives. To focus on effects related to the informativeness of price movements, my analysis up to this point has

---

<sup>25</sup>It may be interesting to examine the properties of aggregate disposition behavior when there is richer variation among investor types, say some disposition investors and some anti-disposition investors, but that matter is beyond the scope of this essay.

abstracted away from the rate at which new private information arrives, by measuring time in terms of private-information arrivals. In this subsection, I abstract away from the details of price informativeness and focus instead on effects related to the rate at which new information arrives.

### 3.5.1.1 The Altered Price-Target Model

The only aspect of the original price-target model that I modify is the public-information structure.

It is no longer the case that a noisy public signal about fundamental value arrives each period. Instead, at each date  $t \geq 1$ , if  $\mathbb{E}_{t-1}^M[V] \neq V$ , the true fundamental value  $V$  will be publicly revealed with some fixed probability  $\theta > 0$ . When the fundamental value  $V$  is publicly revealed, say at time  $\tilde{t}$ , market expectations change so that  $\mathbb{E}_{\tilde{t}}^M[V] = V$ . Since there is no longer any risk and hence no risk discount at time  $\tilde{t}$ , the price (ex-noise) is equal to fundamental value:

$$P_{\tilde{t}} = V$$

### 3.5.1.2 Insights from the Altered Model

This highly stylized altered model reveals a new and important implication of the general price-target framework. Under the modified structure for public information, the PIT knows that in each period, if his signal is true then  $S = V$  will be made public with probability  $\theta$ , while if his signal is false, then there is a zero probability that  $S$  will be made public. Every time a period elapses without  $S$  being publicly revealed, the PIT lowers his estimate of  $\tau$ . If nothing is revealed in period  $t$ , the PIT's (Bayesian) updated estimate of the probability that his signal was true,  $\tau$ , will be

$$\tau_{t+1} = \frac{(1 - \theta)\tau_t}{1 - \theta\tau_t}$$



and  $\{\tau_t\}_{t=0}^{\bar{t}}$  is a strictly decreasing sequence:

$$\begin{aligned}\tau_{t+1} &= \frac{(1-\theta)\tau_t}{1-\theta\tau_t} \\ &= \left(\frac{(1-\theta)}{1-\theta\tau_t}\right)\tau_t \\ &< \tau_t\end{aligned}$$

Furthermore, for all  $t > 0$ , we get the bound

$$\begin{aligned}\tau_{t+1} &= \frac{(1-\theta)\tau_t}{1-\theta\tau_t} \\ &< \left(\frac{(1-\theta)}{1-\theta\tau_0}\right)\tau_t \\ &< \left(\frac{(1-\theta)}{1-\theta\tau_0}\right)^t \tau_0\end{aligned}$$

so the sequence  $\{\tau_t\}_{t=0}^{\bar{t}}$  decays towards zero at a rate no less than one geometric in  $\frac{(1-\theta)}{1-\theta\tau_0}$ .

Intuitively, the absence of price movement now conveys information, and this qualitative result can be generalized to the original price-target model. Easley and O'Hara (1992) examine a similar question about how investors could learn from the absence of trading activity.<sup>26</sup> An important conclusion of their analysis is that investors can learn something about the arrival of new information from trading volume. Although a full treatment of the matter is beyond the scope of this essay, incorporating trading volume into the price-target framework is a promising direction for future research.

### 3.5.2 Comment on the Binary Signal

Stated informally, the central task of the PIT in each period is to determine what true information he knows that the market doesn't know. This task has two distinct components: 1) assessing what initially private information is true, and 2) assessing what initially private (true) information is still private. The first component basically corresponds to the PIT updating his beliefs about the precision of his original signal, while the second compo-

---

<sup>26</sup>The empirical work of Dufour and Engle (2000) also addresses an analogous question.

ment corresponds to the PIT updating his beliefs about the magnitude of the expected price change—relative to the current market price—implied by his initial signal (using his updated beliefs about its precision where applicable). The binary true/false structure for the PIT’s private signal captures these two components in a particularly tractable way, but alternative structures can accomplish the same end.

### 3.5.3 Potential Reconciliation with Prospect Theory

Information and learning are the ultimate focus and foundation of the price-target model. The model does not accommodate *completely* arbitrary PIT preferences, but it imposes very minimal restrictions. The PIT’s utility must be an increasing function of trading profits, and the PIT must be at least slightly risk averse so that he will not hold an excess position in the stock when the expected return is sufficiently small. The price-target model does not require prospect-theoretic PIT preferences to explain the empirical results on disposition behavior, but the model could potentially be compatible with prospect-theoretic PIT preferences.

Prospect-theoretic preferences are described using a “value function” that is defined over gains and losses measured relative to some reference point. This value function, which I will denote by  $v(\cdot)$ , has two key characteristics. First, the value function  $v(\cdot)$  is not differentiable at 0 (more specifically,  $\lim_{\epsilon \rightarrow 0} (v'(-\epsilon) - v'(\epsilon)) > 0$ , which implies greater sensitivity to small losses than to small gains), but  $v'(x) > 0 \forall x \neq 0$ , so there is no conflict with the requirement that the PIT’s utility must be an increasing function of trading profits. The potential incompatibility arises from the second key characteristic of  $v(\cdot)$ , namely that  $v''(x) < 0$  for  $x > 0$ , but  $v''(x) > 0$  for  $x < 0$ . While the price-target model may not actually accommodate prospect-theoretic PIT preferences, the convexity of the value function over the “losses” region does necessarily imply this.

In the carefully circumscribed experimental settings that Kahneman and Tversky considered when developing prospect theory, the appropriate reference point and the payoff to be compared to that reference point could be clearly and unambiguously defined. However, in the context of an investor’s trading decisions, appropriately defining gains and losses is not at

all straight-forward. In static settings, the convention of setting the reference point equal to initial purchase price is fairly innocuous, but extensions to dynamic models require assumptions about how people update their reference points over time. If we suppose that the PIT uses his most recent expectation of fundamental value as a reference point, the convexity of the value function over the “losses” region becomes far less problematic. Provided that some other mechanism prevents the PIT from taking arbitrarily large positions (e.g., finite wealth and borrowing constraints), risk aversion only becomes essential in the price-target model when the PIT’s expectation of fundamental value is very close to the market price. Recall that the value function has the property that  $\lim_{\epsilon \rightarrow 0} (v'(-\epsilon) - v'(\epsilon)) > 0$ . In a neighborhood of zero, second-derivative curvature vanishes, but the difference in the slope of the value function remains, so we get local concavity precisely when it would matter for the price-target model.

Ultimately, the details of how prospect-theoretic preferences could fit into the price-target model are far less important than the general point that belief-based theories and preference-based theories are not in competition, but rather are complementary. Precisely because the two types of theories differ so substantially from one another, there are rich opportunities to enhance each using insights and tools from the other.

### 3.6 Conclusion

The price-target model developed in this essay addresses a new dimension of how investors learn from market prices, namely how investors learn about whether they possess private information. This model helps to explain many observed stylized facts about individuals’ trading behavior in asset markets that existing theories based on investor preferences could not accommodate. Most notably, the price-target model provides a theory of why disposition behavior among different assets varies systematically with the characteristics of the underlying asset market. The model also suggests several novel empirical implications, and I develop general theoretical results which establish that such implications about disposition behavior are indeed testable.

## Chapter 4

# Appendices

### 4.1 Chapter 1 Appendix

#### 4.1.1 Exploratory Trading Model Details

##### 4.1.1.1 Solving the Baseline Exploratory Trading Model

Let  $s_t$  denote the sign of  $q_t$ .

**Solving the Model: Period 2** If  $\varphi = 0$ , the HFT's optimal choice is to not submit an aggressive order in period 2, or equivalently, to set  $|q_2| = 0$ . If  $\varphi \neq 0$ , then it is optimal for the HFT to set  $s_2 = \varphi$  (unless the optimal  $|q_2|$  is zero), so we only need to determine the optimal magnitude,  $|q_2|$ . Because  $\pi_2$  is linear in  $|q_2|$  when  $s_2$  is held fixed, we can restrict attention to corner solutions (0 or  $N$ ) for the optimal choice of  $|q_2|$  without loss of generality. Note that if  $q_2 = 0$ , then  $\pi_2 = 0$ , regardless of the values of  $\varphi$  and  $\Lambda$ .

Suppose that the HFT sets  $|q_2| = N$ . Without loss of generality, assume that  $s_2 = \varphi \neq 0$ . The HFT's period-2 profits are given by

$$\tilde{\pi}_2 = \begin{cases} N(1 - \alpha) & \text{if } \Lambda = U \\ -N\alpha & \text{if } \Lambda = A \end{cases}$$

where the tilde on  $\tilde{\pi}_2$  denotes the fact that the HFT's choice of  $q_2$  does not condition on the

value of  $\Lambda$ .

**HFT Does Not Know  $\Lambda$**  If the HFT does not know the value of  $\Lambda$ , then in the case where  $\varphi \neq 0$ , the HFT's expected period-2 profit if he sets  $|q_2| = N$  is

$$\begin{aligned}\mathbb{E}[\tilde{\pi}_2 | \varphi \neq 0, |q_2| = N] &= uN(1 - \alpha) - (1 - u)N\alpha \\ &= (u - \alpha)N\end{aligned}$$

Taking expectations with respect to  $\varphi$ , we find that the *ex-ante* expectation of  $\tilde{\pi}_2$  when the HFT sets  $|q_2| = N$  (and  $s_2 = \varphi$ ) is given by

$$\mathbb{E}[\tilde{\pi}_2 | |q_2| = N] = v(u - \alpha)N$$

When  $u - \alpha < 0$ , if the HFT did not know  $\Lambda$ , he would set  $q_2 = 0$  rather than  $|q_2| = N$ . Hence the *ex-ante* expectation of  $\tilde{\pi}_2$  is

$$\mathbb{E}[\tilde{\pi}_2] = \max\{v(u - \alpha)N, 0\}$$

**HFT Knows  $\Lambda$**  Next, if the HFT *does* know the value of  $\Lambda$ , then he will set  $|q_2| = N$  (and  $s_2 = \varphi$ ) only when  $\Lambda = U$  and  $\varphi \neq 0$ . Denoting the HFT's period-2 profits from this strategy by  $\hat{\pi}_2$ , we find

$$\begin{aligned}\mathbb{E}[\hat{\pi}_2 | \varphi \neq 0] &= u(1 - \alpha)N \\ &= (u - \alpha)N + \alpha(1 - u)N \\ \mathbb{E}[\hat{\pi}_2] &= vu(1 - \alpha)N \\ &= v(u - \alpha)N + v\alpha(1 - u)N\end{aligned}$$

Note that

$$\mathbb{E}[\hat{\pi}_2] > \max\{v(u - \alpha)N, 0\}$$

so the HFT's expected period-2 profits are strictly greater when he knows  $\Lambda$  than when he doesn't know  $\Lambda$ .

**Solving the Model: Period 1** At the start of period 1, the HFT knows neither  $\varphi$  nor  $\Lambda$ , but he faces the same trading costs,  $\alpha$ , as he does in period 2. Consequently, the HFT's expected direct trading profits from a period-1 aggressive order are negative:

$$\begin{aligned}\mathbb{E}[\pi_1|q_1] &= \mathbb{E}[|q_1|(s_1 y - \alpha)|s_1, q_1] \\ &= |q_1| s_1 \mathbb{E}[y] - \alpha |q_1| \\ &= -\alpha |q_1|\end{aligned}$$

The second equality relies on the assumptions that  $\varphi$  and  $\Lambda$  (and hence  $y$ ) are independent of  $s_1$  and  $q_1$ , while the final equality uses the fact that  $\mathbb{E}[y] = 0$ .

Since there is no noise in this baseline model, the HFT learns  $\Lambda$  perfectly from any aggressive order that he places in the first period with  $|q_1| \geq 1$ . An aggressive order of size greater than one yields no more information about  $\Lambda$  than a one-contract aggressive order in this setting, but the larger aggressive order incurs additional expected losses. Thus without loss of generality, we can restrict attention to the case of  $q_1 = 0$  and the case of  $|q_1| = 1$ .

If the HFT sets  $q_1 = 0$ , he neither learns  $\Lambda$  nor incurs any direct losses in period 1, so his total expected profits are simply

$$\begin{aligned}\mathbb{E}[\pi_{total}|q_1 = 0] &= \mathbb{E}[\tilde{\pi}_2] \\ &= \max\{v(u - \alpha)N, 0\}\end{aligned}$$

Alternatively, if the HFT sets  $|q_1| = 1$ , his total expected profits are given by

$$\begin{aligned}\mathbb{E}[\pi_{total}|q_1| = 1] &= -\alpha |q_1| + \mathbb{E}[\hat{\pi}_2] \\ &= vu(1 - \alpha)N - \alpha\end{aligned}$$

#### 4.1.1.2 Remark on the Sequence of Events

The central results of the model would not change if the HFT observed the signal of future aggressive order-flow *before* deciding whether to engage in exploratory trading, rather than observing it after deciding. However, the sequence of events outlined in section 1.2, in which the HFT must choose whether or not to explore before he observes  $\varphi$ , is more appropriate from an empirical perspective. For the HFT to learn about the liquidity state after he submits an aggressive order, he must wait for 1) his order to reach the market and execute, 2) information about that execution to reach other traders, 3) other traders to decide what to do, 4) other traders' decisions to reach the market, and 5) information about the market response to get back to him. Of these five steps, (1), (2), (4) and (5) each take approximately 3 – 4 milliseconds, and (3) takes considerably longer, perhaps 3 – 20 milliseconds, for an overall total of 15 – 40 milliseconds. An HFT who has already done his exploration will be able to take advantage of predictable aggressive order-flow long before an HFT who only engages in exploratory trading after seeing an order-flow signal.

#### 4.1.1.3 Solving the Model of Section 1.2.3

If the HFT places an order in the first period, it follows immediately from the baseline model results that his expected total profits are given by

$$\mathbb{E}[\pi_{total} | |q_1| = 1] = Nvu(1 - \alpha) - \alpha$$

However, the HFT's expected profits if he does not place an order in period 1 are higher than in the baseline model, because the HFT now learns something from the depth changes following the other trader's aggressive order. If resting depth weakly replenishes after that order, the HFT learns with certainty that the liquidity state is accommodating (i.e.,  $\Lambda = A$ ), so the HFT will not submit an aggressive order in period 2, and his total profits will be zero. Alternatively, if resting depth further depletes following the aggressive order in period

1 (denote this event by  $g_1$ ), we have

$$\begin{aligned}
\mathbb{P}\{\Lambda = U|g_1\} &= \frac{\mathbb{P}\{\Lambda = U, \text{ and } g_1\}}{\mathbb{P}\{g_1\}} \\
&= \frac{\mathbb{P}\{g_1|\Lambda = U\} \mathbb{P}\{\Lambda = U\}}{\mathbb{P}\{g_1|\Lambda = U\} \mathbb{P}\{\Lambda = U\} + \mathbb{P}\{g_1|\Lambda = A\} \mathbb{P}\{\Lambda = A\}} \\
&= \frac{1 * \mathbb{P}\{\Lambda = U\}}{1 * \mathbb{P}\{\Lambda = U\} + \rho * \mathbb{P}\{\Lambda = A\}} \\
&= \frac{u}{u + \rho(1 - u)}
\end{aligned}$$

It follows immediately from the analogous result in the baseline model that the HFT's expected period-2 profits are given by

$$\mathbb{E}[\pi_2|AO \text{ by someone else}] = \max \left\{ Nv \left( \frac{u}{u + \rho(1 - u)} - \alpha \right), 0 \right\}$$

Since the HFT does not place an order in the first period, his expected total profits equal his expected period-2 profits. Overall, then, the HFT's expected total profit if he observes an aggressive order placed by someone else in period 1 but does not place an aggressive order himself, is given by

$$\mathbb{E}[\pi_{total}|AO \text{ by someone else}] = \max \left\{ Nv \left( \frac{u}{u + \rho(1 - u)} - \alpha \right), 0 \right\}$$

#### 4.1.2 Measuring Aggressive Order Profitability

Calculating round-trip profits using a first-in-first-out (“FIFO”) or last-in-first-out (“LIFO”) approach is not a useful way to measure the profitability of individual aggressive orders. Even the most aggressive HFTs engage in some passive trading, so a FIFO/LIFO-round-trip measure would either confound aggressive trades with passive trades, or require some arbitrary assumption to distinguish between inventory acquired passively and inventory acquired aggressively (on top of the already-arbitrary assumption of FIFO or LIFO). A second, more general problem is that a measurement scheme based on inventory round-trips will always combine at least two orders (an entry and an exit), so such measurement schemes do not



actually measure the profitability of *individual* aggressive orders.

In this appendix subsection, I provide rigorous justification for the claim that the average expected profit from an aggressive order in the E-mini market equals the expected favorable price movement, minus trading/clearing fees and half the bid-ask spread. After presenting the formal proof, I discuss details of empirically estimating expected favorable price movement.

#### 4.1.2.1 Preliminaries

Trading/clearing fees apply equally to both passively and aggressively traded E-mini contracts, so to simplify the exposition, I will initially ignore these fees. Similarly, I make the simplifying assumption that the bid-ask spread is constant, and identically equal to one tick; for the E-mini market, this assumption entails minimal loss of generality.

In the E-mini market, the profitability of individual aggressive orders can be considered in isolation from passive orders. Because E-mini contracts can be created directly by buyers and sellers, a trader's net inventory position does not constrain his ability to participate in a given trade<sup>27</sup>. As long as he can find a buyer, a trader who wishes to sell an E-mini contract can always do so, regardless of whether he has a preexisting long position. More generally, if a trader enters a position aggressively then exits it passively, he could have conducted the passive transaction even if he hadn't engaged in the preceding aggressive transaction. While a desire to dispose of passively-acquired inventory might *motivate* a trader to submit an aggressive order, the question of underlying motivation is distinct from the question of whether the aggressive order was directly profitable.

#### 4.1.2.2 Formal Argument

With these preliminaries established, I turn to the rigorous argument. Consider a trader who executes  $J$  aggressive sell orders of size one, and  $J$  aggressive buy orders of size one, for some large  $J$ . Following the remarks above, the trader's passive transactions can be ignored. Let

---

<sup>27</sup>The one exception would arise in the extremely rare event that a trader who did not qualify for a position-limit exemption held so many contracts (either long or short) that his inventory after the trade would exceed the position limit of 100,000 E-mini contracts. For HFTs, this minor exception can safely be ignored.

the average direction-normalized price change after these aggressive orders be  $\tilde{\vartheta} \equiv \vartheta \left( \frac{2J}{2J-1} \right)$  ticks for some  $\vartheta$  that does not depend on  $J$ .

First, suppose that the trader always submits an aggressive sell after an aggressive buy, and always submits an aggressive buy after an aggressive sell. Without loss of generality, assume that the trader's first aggressive order is a buy. The trader's combined profit from all  $2J$  aggressive orders is

$$\begin{aligned}
\pi_{2J} &= -a_1 + b_2 - a_3 + b_4 - \dots - a_{2J-1} + b_{2J} \\
&= -a_1 + (a_2 - 1) - a_3 + (a_4 - 1) - \dots - a_{2J-1} + (a_{2J} - 1) \\
&= -a_1 + a_2 - a_3 + a_4 - \dots - a_{2J-1} + a_{2J} - J \\
&= -a_1 + (a_1 + \zeta_{b,1}) - (a_2 + \zeta_{s,2}) + (a_3 + \zeta_{b,2}) - \dots \\
&\quad \dots - (a_{2J-2} + \zeta_{s,J}) + (a_{2J-1} + \zeta_{b,J}) - J \\
&= \sum_{i=1}^J (a_{2i-1} + \zeta_{b,i}) - \sum_{j=2}^J (a_{2j-2} + \zeta_{s,j}) - a_1 - J \\
&= \sum_{i=1}^J a_{2i-1} - \left( a_1 + \sum_{j=1}^{J-1} a_{2j} \right) + \sum_{i=1}^J \zeta_{b,i} - \sum_{j=2}^J \zeta_{s,j} - J
\end{aligned}$$

where  $a_k$  and  $b_k$  respectively denote the prevailing best ask and best bid at the time the  $k$ th aggressive order executes,  $\zeta_{b,r}$  denotes the change in midpoint price following the  $r$ th aggressive buy order, and  $\zeta_{s,r}$  denotes the change in midpoint price following the  $r$ th aggressive sell order. Note that  $\vartheta \equiv \frac{1}{2J} \left( \sum_{r=1}^J \zeta_{b,r} + \sum_{r=1}^J (-\zeta_{s,r}) \right)$ .

Next, taking expectations, we find

$$\begin{aligned}
\mathbb{E}[\pi_{2J}] &= \sum_{i=1}^J \mathbb{E}[a_{2i-1}] - \left( \mathbb{E}[a_1] + \sum_{j=1}^{J-1} \mathbb{E}[a_{2j}] \right) \\
&\quad + \sum_{i=1}^J \mathbb{E}[\zeta_{b,i}] - \sum_{j=2}^J \mathbb{E}[\zeta_{s,j}] - J \\
&= J\mathbb{E}[a_1] - \mathbb{E}[a_1] - (J-1)\mathbb{E}[a_1] + J\mathbb{E}[\tilde{\vartheta}] - (J-1)(-\mathbb{E}[\tilde{\vartheta}]) - J \\
&= (2J-1)\mathbb{E}[\tilde{\vartheta}] - J \\
&= (2J-1)\left(\mathbb{E}[\vartheta] \frac{2J}{2J-1}\right) - J \\
&= J(2\mathbb{E}[\vartheta] - 1)
\end{aligned}$$

where the second equality uses the assumption that midpoint prices follow a martingale with respect to their natural filtration, together with the assumption of a constant bid-ask spread. From the final equality above, it follows immediately that the trader's average expected profit on an individual aggressive order is given by

$$\frac{1}{2J}\mathbb{E}[\pi_{2J}] = \mathbb{E}[\vartheta] - \frac{1}{2}$$

Finally, note that none of the calculations above relied on the assumption that the aggressive orders alternated between buys and sells (this only simplified the notation). It follows immediately from grouping together multiple aggressive orders of the same sign that the result would hold for orders of varying sizes, provided that the overall aggressive buy and aggressive sell volumes were equal.

Under the usual regularity conditions, as  $J \rightarrow \infty$ ,  $\tilde{\vartheta} \rightarrow_{A.S.} \lim_{J \rightarrow \infty} \mathbb{E}[\tilde{\vartheta}] = \mathbb{E}[\vartheta]$ .  $\square$

**Independent Importance of the Result** Establishing a meaningful technique to estimate the performance of individual aggressive orders was merely a necessary stepping stone for a detailed analysis of HFTs' performance on aggressive orders, but to the best of my knowledge, I am the first to propose and rigorously justify this technique. This technique may have broad

applications for academics, regulators and practitioners alike.

#### 4.1.2.3 Obtaining Unbiased Estimates

Recall that the discussion in section 1.4.1 implied that we can estimate the profitability of an HFT's aggressive order using the (direction-normalized) accumulated price-changes following that aggressive order out to some time past the HFT's maximum forecasting horizon. If we choose too short an accumulation window, the resulting estimates of the long-run direction-normalized average price changes following an HFT's aggressive orders will be biased downward. This enables us to empirically determine an adequate accumulation period by calculating cumulative direction-normalized price changes over longer and longer windows until their mean ceases to significantly increase

Market activity varies considerably in its intensity throughout a trading day, so event-time, which I measure in terms of aggressive order arrivals, provides a more uniform standard for temporal measurements than does clock-time. Empirically, an accumulation period of about 30 aggressive orders suffices to obtain unbiased estimates of the price movement following an HFT's aggressive order, but I consider results for an accumulation period of 50 aggressive orders to allow a wide margin for error. The mean direction-normalized price changes following individual HFTs' aggressive orders does not differ significantly for accumulation periods of 50, 200, or 500 aggressive orders, even if we distinguish aggressive orders by size. The same holds true for aggressive orders placed by non-HFTs. Using too long an accumulation period will not bias the estimates, but it will introduce unnecessary noise, so I opt for an accumulation period of 50 aggressive orders.

As I discuss at greater length in section 1.4.3, future price movements are moderately predictable from past aggressive order flow and orderbook activity, but only at very short horizons. Of the variables that meaningfully forecast future price changes, the direction of aggressive order flow is by far the most persistent, but even its forecasting power diminishes to nonexistence for price movements more than either about 12 aggressive orders or 200 milliseconds in the future. The adequacy of a 30+ aggressive order accumulation period is

entirely consistent with these results.

As a simple empirical check on the validity of direction-normalized cumulative price changes as a proxy for the profitability of aggressive orders, I use each HFT's explicit overall profits and passive trading volume, together with the profits on aggressive orders as measured by the proxy, to back out the HFT's implicit profit on each passively traded contract. The resulting estimates of HFTs' respective profits from passive transactions are all plausible from a theoretical perspective, and are comparable to non-HFTs' implicit performance on passive trades.

### 4.1.3 Benchmark Regression Results

In this appendix subsection, I present and discuss results from regression (1.8) of section 1.4.3.

Recall that for each trading day in my sample, I regress the cumulative price-change (in dollars) between the aggressive orders  $k$  and  $k + 50$ , denoted  $y_k$ , on the following variables: changes in resting depth between aggressive orders  $k - 1$  and  $k$  at each of the six price levels within two ticks of the best bid or best ask, the signs of aggressive orders  $k - 1$  through  $k - 4$ , and the signed executed quantities of aggressive orders  $k - 1$  through  $k - 4$ . For symmetry, I adopt the convention that sell depth is negative, and buy depth is positive, so that an increase in buy depth has the same sign as a decrease in sell depth. I estimate the equation

$$\begin{aligned} y_k &= z_{k-1}\Gamma + \epsilon_k \\ &:= \gamma_1 d_{k-1}^1 + \dots + \gamma_6 d_{k-1}^6 + \\ &\quad \gamma_7 \text{sign}_{k-1} + \dots + \gamma_{10} \text{sign}_{k-4} + \\ &\quad \gamma_{11} q_{k-1} + \dots + \gamma_{14} q_{k-4} + \epsilon_k \end{aligned}$$

where  $d_{k-1}^r$  denotes the change in resting depth at price level  $r$  ( $r = 3$  corresponds to the best bid,  $r = 4$  corresponds to the best ask),  $\text{sign}_l$  denotes the sign of aggressive order  $l$ , and  $q_l$  denotes the signed executed quantity of aggressive order  $l$ .

Table 1.8 summarizes the estimates from the regression above, computed over my entire

sample. All of the variables are antisymmetrical for buys and sells, and so have means extremely close to zero, but the mean magnitudes in the rightmost column of Table 1.8 provide some context for scale.

Table 1.8: Estimates from the Benchmark Regression

	Coefficient $\times 10^3$	Robust t-Statistic	Variable Avg. Magnitude
$d_{k-1}^{best\ bid-2}$	-0.90	-1.02	4.13
$d_{k-1}^{best\ bid-1}$	-2.08	-4.29	10.8
$d_{k-1}^{best\ bid}$	1.13	4.94	23.1
$d_{k-1}^{best\ ask}$	1.11	4.97	23.4
$d_{k-1}^{best\ ask+1}$	-2.03	-4.24	11.2
$d_{k-1}^{best\ ask+2}$	-1.60	-1.90	4.44
$sign_{k-1}$	1186	33.3	1
$sign_{k-2}$	753	20.2	1
$sign_{k-3}$	544	14.6	1
$sign_{k-4}$	472	13.4	1
$q_{k-1}$	4.09	9.29	12.6
$q_{k-2}$	2.66	6.59	12.6
$q_{k-3}$	1.85	4.66	12.6
$q_{k-4}$	1.16	2.98	12.6

Comparable results obtain using as few as two lags of aggressive order sign and signed quantity. Linear forecasts of  $y_k$  do not benefit appreciably from the inclusion of data on aggressive orders before  $k - 4$ , or on changes in resting depth prior to aggressive order  $k - 1$ .

Because the price-change  $y_k$  is not normalized by the sign of the  $k$ th aggressive order, it has an expected value of zero, so I do not include a constant term in the regression. Including a constant term in the regression has negligible effect on the results. Although the last several aggressive order signs do offer rather remarkable explanatory power, the respective distributions of resting depth changes and executed aggressive order quantities have much heavier tails than the distribution of order sign, so price forecasts are substantially improved by the inclusion of these variables.

The positive coefficients on the lagged aggressive order variables and on the depth changes at the best bid and best ask are consistent with the general intuition that buy orders portend price increases, and sell orders portend price decreases. The negative coefficients on depth changes at the outside price levels require slightly more explanation.

Because the E-mini market operates according to strict price and time priority, a trader who seeks priority execution of his passive order will generally place that order at the best bid (or best ask); however, if the trader believes that an adverse price movement is imminent, he will place his order at the price level that he expects to be the best bid (ask) following the price change. It is relatively uncommon for prices to change immediately after an aggressive order in the E-mini market, but when prices do change, it is extremely rare during regular trading hours for the change to exceed one tick. As a result, the expected best bid (ask) following a price change is typically one tick away from the previous best, so it is not surprising that (e.g.) an increase in resting depth one tick below the best bid tends to precede a downward price change. These features of the E-mini market also shed some light on why changes in depth more than one tick away from the best (i.e.,  $d_{k-1}^{best\ bid-2}$  and  $d_{k-1}^{best\ ask+2}$ ) are not significant predictors of future price movements.<sup>28</sup>

---

<sup>28</sup>Similarly, this line of reasoning helps to explain why changes in resting depth prior to aggressive order  $k - 1$  do not help to forecast price changes after the  $k$ th aggressive order.

#### 4.1.4 Supplemental Tables of Empirical Results

##### 4.1.4.1 Results for Extended Values of $\bar{q}$

Table 1.9 presents numerical estimates for extended values of  $\bar{q}$ . The lefthand side of Table 1.9 presents averages of the estimated additional gross earnings per contract on aggressive orders of submitted size greater than  $\bar{q}$  explained by regression (1.10) in excess of that explained by regression (1.9) for the indicated trader type. The abbreviation “EE” stands for “everyone else.” Numbers reported for the A-HFTs are averages over the estimates for individual A-HFTs. The right-hand side of Table 1.9 presents cross-sectional averages of the difference in mean additional gross earnings per contract on aggressive orders of submitted size greater than  $\bar{q}$  explained by regression (1.10) in excess of that explained by regression (1.9), between the A-HFTs and the indicated groups. The membership of “everyone else” depends upon the particular A-HFT being excluded, and the numbers reported or implicitly subtracted for “everyone else” are averaged over these slightly different groups; the same is true for the group “other HFTs.” Units are cents per contract, and confidence intervals are constructed by bootstrap.

##### 4.1.4.2 Aggressive Order Characteristics Across Trader Types

Table 1.10 breaks down average aggressive order performance by order size and trader type. Table 1.11 summarizes cumulative empirical distributions of aggressive trading activity for each type of trader.



Table 1.9: Additional Explained Performance for Extended Values of  $\bar{q}$ 

$\bar{q}$	Trader Type	Point Est.	95% CI	A-HFTs vs.	Point Est.	95% CI
25	A-HFTs	0.624	(0.437, 0.806)	EE	0.450	(0.262, 0.639)
	EE	0.175	(0.135, 0.213)	Other HFTs	0.360	(0.171, 0.554)
30	A-HFTs	0.697	(0.520, 0.894)	EE	0.551	(0.370, 0.750)
	EE	0.147	(0.106, 0.182)	Other HFTs	0.454	(0.272, 0.660)
35	A-HFTs	0.733	(0.551, 0.920)	EE	0.590	(0.406, 0.781)
	EE	0.143	(0.106, 0.184)	Other HFTs	0.489	(0.299, 0.684)
40	A-HFTs	0.850	(0.646, 1.037)	EE	0.688	(0.483, 0.876)
	EE	0.162	(0.120, 0.205)	Other HFTs	0.580	(0.362, 0.779)
45	A-HFTs	0.850	(0.659, 1.053)	EE	0.688	(0.492, 0.896)
	EE	0.162	(0.119, 0.205)	Other HFTs	0.560	(0.354, 0.776)
50	A-HFTs	1.003	(0.810, 1.219)	EE	0.804	(0.599, 1.019)
	EE	0.199	(0.145, 0.253)	Other HFTs	0.643	(0.435, 0.871)
60	A-HFTs	1.181	(0.985, 1.381)	EE	0.964	(0.755, 1.172)
	EE	0.218	(0.162, 0.272)	Other HFTs	0.786	(0.562, 0.992)
75	A-HFTs	1.073	(0.860, 1.293)	EE	0.902	(0.687, 1.128)
	EE	0.171	(0.112, 0.230)	Other HFTs	0.757	(0.535, 1.003)
90	A-HFTs	1.040	(0.821, 1.242)	EE	0.920	(0.685, 1.151)
	EE	0.120	(0.053, 0.187)	Other HFTs	0.746	(0.498, 0.991)

Table 1.10: Size-Weighted Avg. Performance of Aggressive Orders Below/Between Size Thresholds (Dollars per Contract)

$\bar{q}$	A-HFTs		B-HFTs		Non-HFTs	
	All AOs $\leq \bar{q}$	Incremental AOs	All AOs $\leq \bar{q}$	Incremental AOs	All AOs $\leq \bar{q}$	Incremental AOs
1	3.84	-	4.37	-	1.68	-
5	4.23	4.38	4.56	4.64	1.72	1.75
10	3.49	2.84	4.66	4.85	1.77	1.89
15	3.85	6.39	4.71	4.95	1.79	1.97
20	4.14	4.95	4.77	5.08	1.83	2.06
25	4.41	6.65	4.81	5.14	1.86	2.20
30	4.79	6.79	4.87	5.24	1.90	2.53
35	4.99	7.00	4.88	5.10	1.91	2.40
40	5.28	6.69	4.91	5.14	1.95	2.55
45	5.42	7.04	4.92	5.49	1.95	2.14
50	5.61	6.90	4.95	5.14	1.96	2.00
60	5.87	7.00	4.98	5.30	1.99	2.87
75	6.12	7.20	5.00	5.40	2.03	2.71
90	6.38	7.20	5.01	5.52	2.03	2.16

Table 1.11: Aggressive Orders Below Size Thresholds: Fractions of All Aggressive Orders and Aggressive Volume

$\bar{q}$	A-HFTs		B-HFTs		Non-HFTs	
	% of All AOs	% of Aggr. Volume	% of All AOs	% of Aggr. Volume	% of All AOs	% of Aggr. Volume
1	24.31%	0.40%	39.48%	4.77%	53.79%	5.68%
5	43.74%	1.44%	76.09%	16.13%	83.26%	14.88%
10	54.64%	3.09%	84.10%	23.95%	89.87%	20.68%
15	56.75%	3.54%	88.01%	30.03%	91.57%	23.05%
20	60.82%	4.80%	90.70%	35.82%	93.27%	26.43%
25	62.38%	5.37%	92.36%	40.29%	94.41%	29.32%
30	64.62%	6.39%	94.14%	46.30%	94.98%	31.07%
35	65.82%	7.02%	94.97%	49.56%	95.23%	31.94%
40	68.27%	8.51%	96.03%	54.31%	95.66%	33.69%
45	69.29%	9.20%	96.32%	55.76%	95.80%	34.33%
50	71.07%	10.55%	97.66%	63.29%	97.09%	41.08%
60	73.81%	12.97%	98.54%	69.12%	97.36%	42.66%
75	76.65%	16.01%	99.02%	73.14%	97.64%	44.76%
90	80.68%	21.11%	99.20%	74.86%	97.83%	46.37%

## 4.2 Chapter 2 Appendix

### 4.2.1 Experimental Instructions for the “Choice” Treatment<sup>29</sup>

Code number: \_\_\_\_\_

#### **General information about the Investment Study**

In this study you can earn money by trading assets. The money you earn is real, but the assets are created for this study only. The amount of money you earn will depend on your decisions and on how the prices of the assets that you hold develop. At the end of the study, your earnings will be added to the show-up fee of \$10 and you will be paid in cash before you leave. This study is conducted anonymously, and you and all other participants will be identified only by code numbers. You find your code number at the top left of this page. The instructions for the study are described here. If you have any questions after you have read and heard the instructions, please press the help button or raise your hand and someone will come by and help you. Please, do not talk or otherwise communicate during the study.

#### **12 investment sessions with 11 trading periods each**

The study consists of 12 investment sessions that are independent of one another. This means that what happens in one investment session does not in any way influence what happens in any other investment session. At the beginning of each session you will be given \$120 to use during that investment session. You can use this money to trade in five assets, labeled A, B, C, D, and E. You can also hold cash, which is called asset F. The difference between the assets A, B, C, D, and E, and cash (asset F) is that the value of cash will be constant whereas the prices of the other assets will change each period. Each investment session consists of 11 trading periods. You can buy and sell assets in every trading period.

---

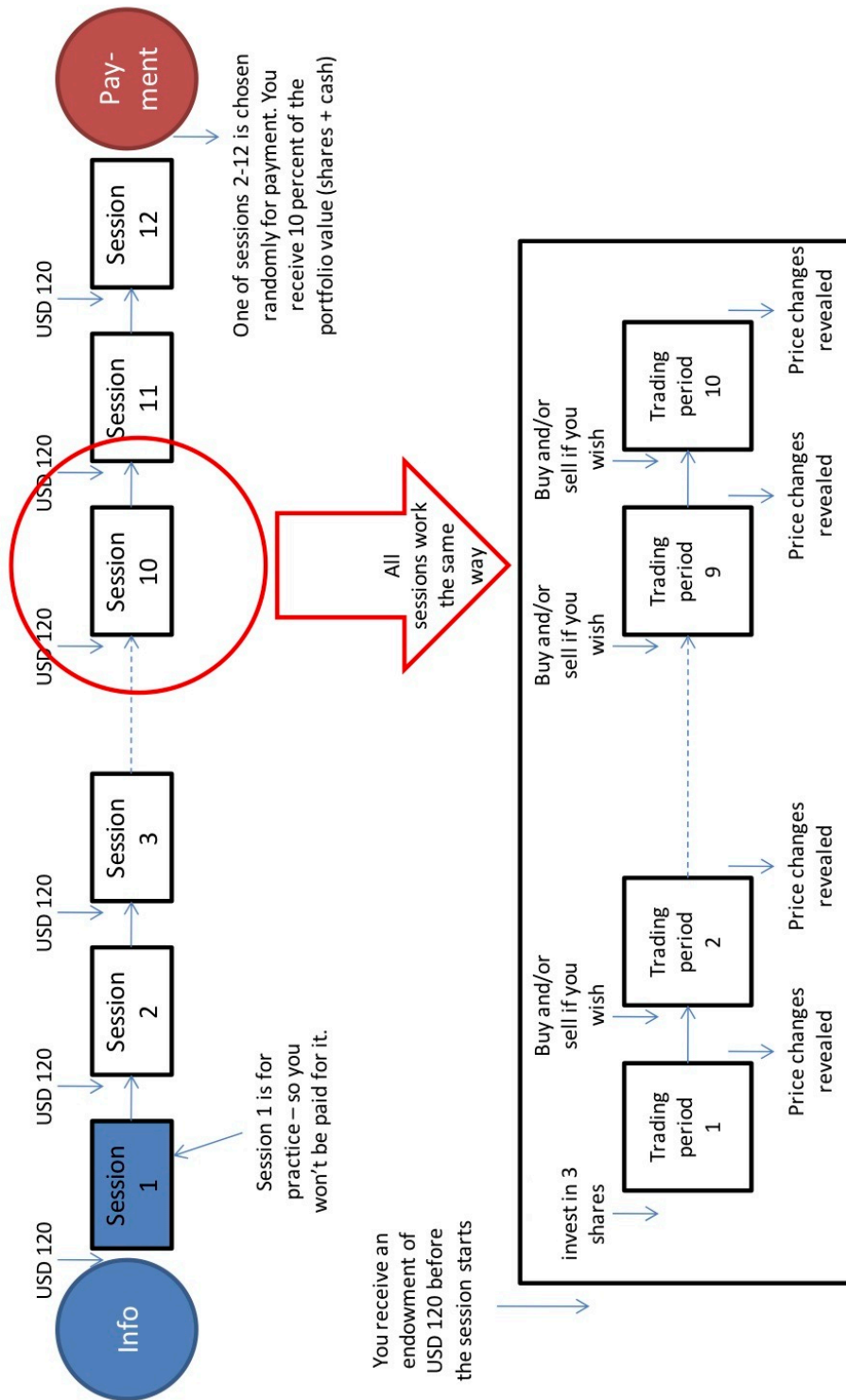
<sup>29</sup>The instructions for “writing” and “no writing” were the same, except that the “writing” treatment included the instructions under the heading “Paper Record,” and the “no writing” treatment did not.

## **Trading**

At the start of each investment session, before the first trading period, you have to choose three assets to buy. You have to invest your total endowment of USD 120 in equal proportions in the three assets. That means that you choose three of the six assets (three of A, B, C, D, E, and F) and put USD 40 in each of the three. When you have made your choice and executed your trade, you will see on the screen how the prices of the all of the assets change. You have then reached the first trading period and you can buy and sell assets. Only before the first trading period of each investment session will you be restricted to put exactly USD 40 in exactly three distinct assets. In all other trading periods, the only restriction will be that you cannot hold a negative quantity of any asset (including cash). You can sell assets that you own, you can buy more of the assets you already hold, and you can buy assets that you do not yet own. When you buy a share of an asset, the price of that share is deducted from your cash holdings; likewise, when you sell a share of an asset, the price of that share is added to your cash holdings. Subject to the prohibition against holding negative quantities, you can hold the assets in any proportions that you like, and you can hold any number (1-6) of assets in any combination that you desire.

## **Your earnings from the study**

The first investment session is for practice only and will not impact your earnings. At the end of the study we will randomly choose an investment session between 2 and 12, and you will receive 10 percent of the total value of your holdings at the end of that investment session. Since you don't know which of the investment sessions that will be used for payment it is important that you do your best to earn money in all sessions. You will be paid all the money you earn (including the show-up fee of \$10) before you leave.



### Quiz 1 about the investment study

1. How many investment sessions will the study have?

**Answer:** It will have \_\_\_\_\_ investment sessions.

2. Each investment session consists of several trading periods. How many trading periods are there in one investment session?

**Answer:** There are \_\_\_\_\_ trading periods in every investment session

3. How much money will you be endowed with at the start of each investment session?

**Answer:** I will have \$ \_\_\_\_\_

4. Before the first trading period you have to choose a certain number of assets to put your endowment of USD 120 in. How many assets do you have to choose? How much money do you have to put in each asset?

**Answer:** I have to choose \_\_\_\_\_ assets and I have to put \$\_\_\_\_\_ in each of those.

5. Are you allowed to also choose cash (asset F) before the first trading period?

**Answer:**      ☐ Yes      ☐ No

6. After the initial decision in each investment session, are you still restricted to only buying a certain number of assets?

**Answer:**

☐ Yes, I can still only buy three assets.

☐ No, I can buy and sell however I want.

7. Does the amount of money that you can invest in investment session 5 depend on what happens in investment session 4 or investment session 6?

**Answer:**

☐ Yes, if I do well in investment session 4 I have more money in investment session 5

☐ No, the investment sessions are independent.

## Asset prices

At the start of each investment session, all assets have the same price: USD 1. After each trading period, the price of each asset (except for cash) changes in a specific random manner. This random process will now be described to you.

For each asset A, B, C, D and E there is a probability of  $x$  percent that the price of the asset will go up, and a probability of  $100-x$  percent that the price of the asset will go down. For a given asset, the number  $x$  is the same in every period in the same investment session, but the number  $x$  is different for each different asset:

- The asset with trend “++” (double plus) has  $x=65\%$ , i.e. in 65 cases out of 100 the price of this share will go up.
- The asset with trend “+” (plus) has  $x=55\%$ , i.e. in 55 cases out of 100 the price of this share will go up.
- The asset with trend “0” (zero) has  $x=50\%$ , i.e. in 50 cases out of 100 the price of this share will go up.
- The asset with trend “-” (minus) has  $x=45\%$ , i.e. in 45 cases out of 100 the price of this share will go up.
- The asset with trend “--” (double minus) has  $x=35\%$ , i.e. in 35 cases out of 100 the price of this share will go up.

You will not be told which of the asset A, B, C, D, and E has which trend. Although the  $x$ -values for each asset remain the same in each period of a given investment session, the  $x$ -values are randomly reassigned among the assets at the start of each new investment session. For example, if asset B has trend “0”, i.e.  $x=50\%$  in the first investment session, this does not mean that asset B is any more or less likely to have  $x=50\%$  in the second investment session or any investment session thereafter.

In every trading period, it is first determined for every asset whether its price is going up or down—i.e., the **direction** of the price change is determined. This happens the way it is



described above. After that, it is determined by how much the price goes up or down—i.e., the **magnitude** of the price change is determined. The price can move by 1, 3 or 5 percent. These possibilities are equally likely, i.e. in a third of the cases, the price will move by 1 percent, in one third it will move by 3 percent and in one third of the cases it will move by 5 percent. These magnitudes are independent across assets and across trading periods. Asset F, cash, is special. For the other assets (A, B, C, D, and E) you don't know which trend they follow, but you know that asset F always is cash. Furthermore, the value of asset F doesn't change. If you put USD 5 in asset F it will be worth USD 5 in all remaining trading periods. For the other assets (A, B, C, D, and E) the value of your holdings will change with the price of the asset.

After some investment sessions you will be asked which asset you think had which trend in the session that you just finished. You have to answer the question to be able to continue. You will receive \$1 for every investment session this question is asked, if you provide the correct answer. Any money you earn from this task will be added to your other earnings and you will get paid before you leave.

## Quiz 2 about the investment study

1. How many assets are there, in any given investment session, which have a probability of 55% of increasing in price after any given period?

**Answer:**      ☐ None      ☐ One      ☐ Two

2. What does it mean that an asset has trend “++” (double plus)?

**Answer:**

- ☐ Its price will go up with probability 50%
- ☐ Its price will go up with probability 65%
- ☐ Its price will go down with probability 65%
- ☐ It is cash

3. Which asset has trend “-“ (minus)?

**Answer:**

- ☐ Always asset B
- ☐ Always asset D
- ☐ It will be different in different investment sessions

Example 4: If, at the end of a investment session, the total value of your holdings of assets A-E is USD 92, and your cash holdings are USD 30, how much money will you earn from that investment session (beyond your show-up fee) if it is chosen for payment?

**Answer:**      ☐ USD 14      ☐ USD 9.5      ☐ USD 12.2      ☐ USD 15

Example 5: How many assets (including cash) can you maximally hold at the same time?

**Answer:** I can hold \_\_\_\_\_ assets at the same time.

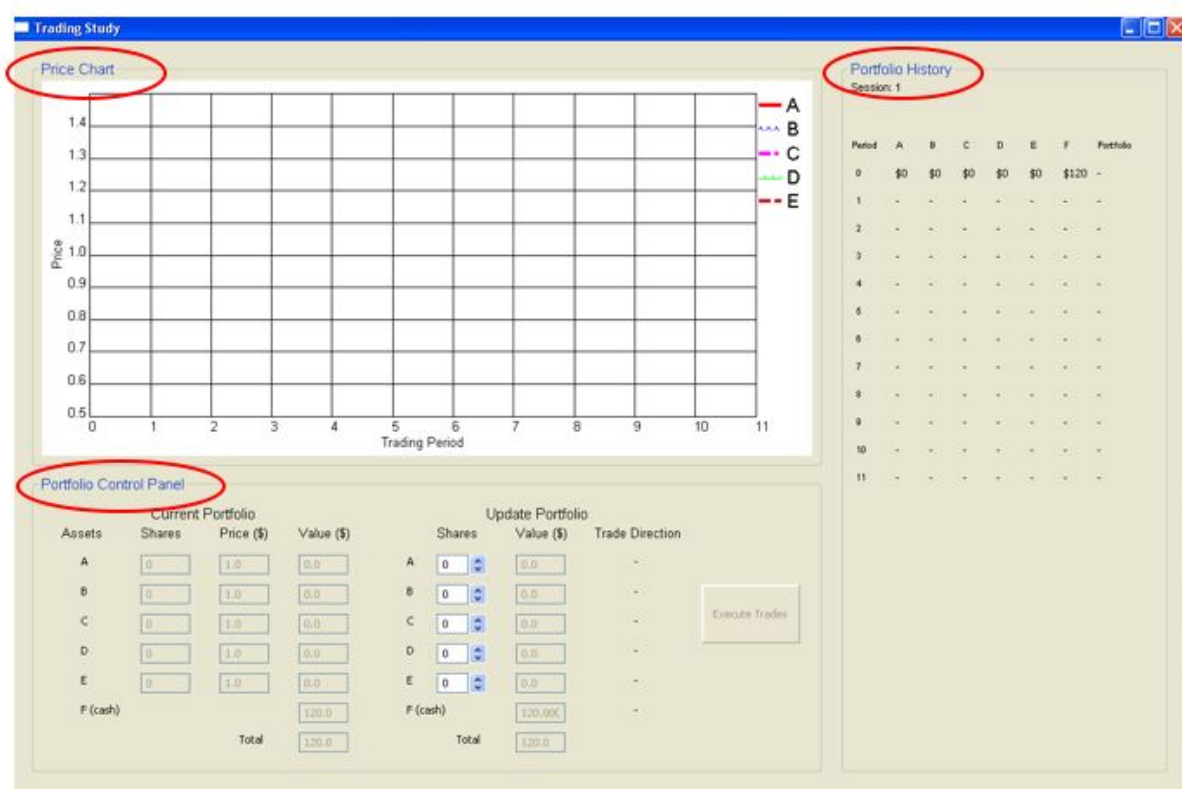
Example 6: What is true about cash? Mark all that apply.

**Answer:**

- ☐ Cash can be held in all periods
- ☐ The value of cash will stay constant
- ☐ Cash is always called asset F
- ☐ All of the above

## Your trading screen

This is what your screen will look like. There are three main parts to the screen: The price chart, the portfolio control panel and the portfolio history.



**1. Price chart:** This chart will be empty when an investment session starts. As you buy and sell and move through the trading periods, the prices of assets A, B, C, D, and E will be visible in the chart.

**2. Portfolio Control Panel:** The left part of this panel is the current portfolio. There you can see what assets you have, how many of each, how much you have in cash, and what the price of each asset is. You make your buying and selling decisions in the right part of the panel, “update portfolio”. Use the arrows to increase or decrease the number of assets.

Whatever money you don't have in any of the assets A, B, C, D or E will automatically be in cash, asset F. Note that when you click the up-arrow, and the number of shares is increasing you are buying. When you click the down-arrow, and the number of shares is decreasing, you are selling. When you click on 'Execute Trades' you will make the changes (if any) that you put into the "update portfolio" part of the panel. You will then also see how the prices change, and if the value of your assets went down or up, i.e. if you gained or lost money from your investments.

**3. Portfolio History:** For your convenience you can check the portfolio history table if you want to be reminded of what your holdings were in the different trading periods. You can see your history for the current investment session but not for past investment sessions.

### Paper Record<sup>30</sup>

You have received a form labeled "Paper Record". On this form you should, in each investment session, circle the three assets that you have in trading period 0. When you have reached trading period 1 you should note on the form if the value of these assets went up or down by circling either the plus (if the value of the asset increased) or the minus (if the value of the asset decreased). This is illustrated below for the case where assets B, C and F were held in trading period 0 and the value of B went up and the value of C went down between trading period 0 and 1.

Investment session 1

Holdings from trading period 0:	A	<b>B</b>	<b>C</b>	D	E	<b>F</b>
Change btw period 0 and 1:	+ / -	<b>+</b> / -	+ / <b>-</b>	+ / -	+ / -	0

---

<sup>30</sup>The instructions under this heading were only given to subjects in the "writing" treatment group.

## **Going forward**

We will now start with investment session 1. As we pointed out before, this session is not for payment but for practice only. When you have finished the first session, please do NOT continue until we tell you to do so. We will wait until everyone has finished investment session 1 and then answer any questions before we move on to investment sessions 2 to 12.

### 4.2.2 Experimental Instructions for the “Assigned” Treatment<sup>31</sup>

Code number: \_\_\_\_\_

#### **General information about the Investment Study**

In this study you can earn money by trading assets. The money you earn is real, but the assets are created for this study only. The amount of money you earn will depend on your decisions and on how the prices of the assets that you hold develop. At the end of the study, your earnings will be added to the show-up fee of \$10 and you will be paid in cash before you leave. This study is conducted anonymously, and you and all other participants will be identified only by code numbers. You find your code number at the top left of this page. The instructions for the study are described here. If you have any questions after you have read and heard the instructions, please press the help button or raise your hand and someone will come by and help you. Please, do not talk or otherwise communicate during the study.

#### **12 investment sessions with 11 trading periods each**

The study consists of 12 investment sessions that are independent of one another. This means that what happens in one investment session does not in any way influence what happens in any other investment session. At the beginning of each session you will be given \$120 to use during that investment session. You can use this money to trade in five assets, labeled A, B, C, D, and E. You can also hold cash, which is called asset F. The difference between the assets A, B, C, D, and E, and cash (asset F) is that the value of cash will be constant whereas the prices of the other assets will change each period. Each investment session consists of 11 trading periods. You can buy and sell assets in every trading period.

#### **Trading**

At the start of each investment session, before the first trading period, you will be randomly allocated three assets to buy. Your total endowment of USD 120 will be invested in equal

---

<sup>31</sup>The instructions for “writing” and “no writing” were the same, except that the “writing” treatment included the instructions under the heading “Paper Record,” and the “no writing” treatment did not.

proportions in the three assets. That means that you will be randomly allocated three of the six assets (three of A, B, C, D, E, and F) with USD 40 in each of the three. After you have executed your trade, you will see on the screen how the prices of the all of the assets change. You have then reached the first trading period and you can buy and sell assets. Only before the first trading period of each investment session will you be you restricted to put exactly USD 40 in exactly three randomly allocated assets. In all other trading periods, the only restriction will be that you cannot hold a negative quantity of any asset (including cash). You can sell assets that you own, you can buy more of the assets you already hold, and you can buy assets that you do not yet own. When you buy a share of an asset, the price of that share is deducted from your cash holdings; likewise, when you sell a share of an asset, the price of that share is added to you cash holdings. Subject to the prohibition against holding negative quantities, you can hold the assets in any proportions that you like, and you can hold any number (1-6) of assets in any combination that you desire.

### **Your earnings from the study**

The first investment session is for practice only and will not impact your earnings. At the end of the study we will randomly choose an investment session between 2 and 12, and you will receive 10 percent of the total value of your holdings at the end of that investment session. Since you don't know which of the investment sessions that will be used for payment it is important that you do your best to earn money in all sessions. You will be paid all the money you earn (including the show-up fee of \$10) before you leave.





### Quiz 1 about the investment study

1. How many investment sessions will the study have?

**Answer:** It will have \_\_\_\_\_ investment sessions.

2. Each investment session consists of several trading periods. How many trading periods are there in one investment session?

**Answer:** There are \_\_\_\_\_ trading periods in every investment session

3. How much money will you be endowed with at the start of each investment session?

**Answer:** I will have \$ \_\_\_\_\_

4. Before the first trading period you are randomly allocated a certain number of assets that your endowment of USD 120 will be put in. How many assets are you allocated? How much money is put in each asset?

**Answer:** I am allocated \_\_\_\_\_ assets and \$\_\_\_\_\_ is put in each of those.

5. Can you be allocated cash (asset F) before the first trading period?

**Answer:**      ☐ Yes      ☐ No

6. After the initial decision in each investment session, are you still restricted to only buying a certain number of assets?

**Answer:**

☐ Yes, I can still only buy three assets.

☐ No, I can buy and sell however I want.

7. Does the amount of money that you can invest in investment session 5 depend on what happens in investment session 4 or investment session 6?

**Answer:**

☐ Yes, if I do well in investment session 4 I have more money in investment session 5

☐ No, the investment sessions are independent.

## Asset prices

At the start of each investment session, all assets have the same price: USD 1. After each trading period, the price of each asset (except for cash) changes in a specific random manner. This random process will now be described to you.

For each asset A, B, C, D and E there is a probability of  $x$  percent that the price of the asset will go up, and a probability of  $100-x$  percent that the price of the asset will go down. For a given asset, the number  $x$  is the same in every period in the same investment session, but the number  $x$  is different for each different asset:

- The asset with trend “++” (double plus) has  $x=65\%$ , i.e. in 65 cases out of 100 the price of this share will go up.
- The asset with trend “+” (plus) has  $x=55\%$ , i.e. in 55 cases out of 100 the price of this share will go up.
- The asset with trend “0” (zero) has  $x=50\%$ , i.e. in 50 cases out of 100 the price of this share will go up.
- The asset with trend “-” (minus) has  $x=45\%$ , i.e. in 45 cases out of 100 the price of this share will go up.
- The asset with trend “--” (double minus) has  $x=35\%$ , i.e. in 35 cases out of 100 the price of this share will go up.

You will not be told which of the asset A, B, C, D, and E has which trend. Although the  $x$ -values for each asset remain the same in each period of a given investment session, the  $x$ -values are randomly reassigned among the assets at the start of each new investment session. For example, if asset B has trend “0”, i.e.  $x=50\%$  in the first investment session, this does not mean that asset B is any more or less likely to have  $x=50\%$  in the second investment session or any investment session thereafter.

In every trading period, it is first determined for every asset whether its price is going up or down—i.e., the **direction** of the price change is determined. This happens the way it is

described above. After that, it is determined by how much the price goes up or down—i.e., the **magnitude** of the price change is determined. The price can move by 1, 3 or 5 percent. These possibilities are equally likely, i.e. in a third of the cases, the price will move by 1 percent, in one third it will move by 3 percent and in one third of the cases it will move by 5 percent. These magnitudes are independent across assets and across trading periods. Asset F, cash, is special. For the other assets (A, B, C, D, and E) you don't know which trend they follow, but you know that asset F always is cash. Furthermore, the value of asset F doesn't change. If you put USD 5 in asset F it will be worth USD 5 in all remaining trading periods. For the other assets (A, B, C, D, and E) the value of your holdings will change with the price of the asset.

After some investment sessions you will be asked which asset you think had which trend in the session that you just finished. You have to answer the question to be able to continue. You will receive \$1 for every investment session this question is asked, if you provide the correct answer. Any money you earn from this task will be added to your other earnings and you will get paid before you leave.

## Quiz 2 about the investment study

1. How many assets are there, in any given investment session, which have a probability of 55% of increasing in price after any given period?

**Answer:**      ☐ None      ☐ One      ☐ Two

2. What does it mean that an asset has trend “++” (double plus)?

**Answer:**

- ☐ Its price will go up with probability 50%
- ☐ Its price will go up with probability 65%
- ☐ Its price will go down with probability 65%
- ☐ It is cash

3. Which asset has trend “-“ (minus)?

**Answer:**

- ☐ Always asset B
- ☐ Always asset D
- ☐ It will be different in different investment sessions

Example 4: If, at the end of a investment session, the total value of your holdings of assets A-E is USD 92, and your cash holdings are USD 30, how much money will you earn from that investment session (beyond your show-up fee) if it is chosen for payment?

**Answer:**      ☐ USD 14      ☐ USD 9.5      ☐ USD 12.2      ☐ USD 15

Example 5: How many assets (including cash) can you maximally hold at the same time?

**Answer:** I can hold \_\_\_\_\_ assets at the same time.

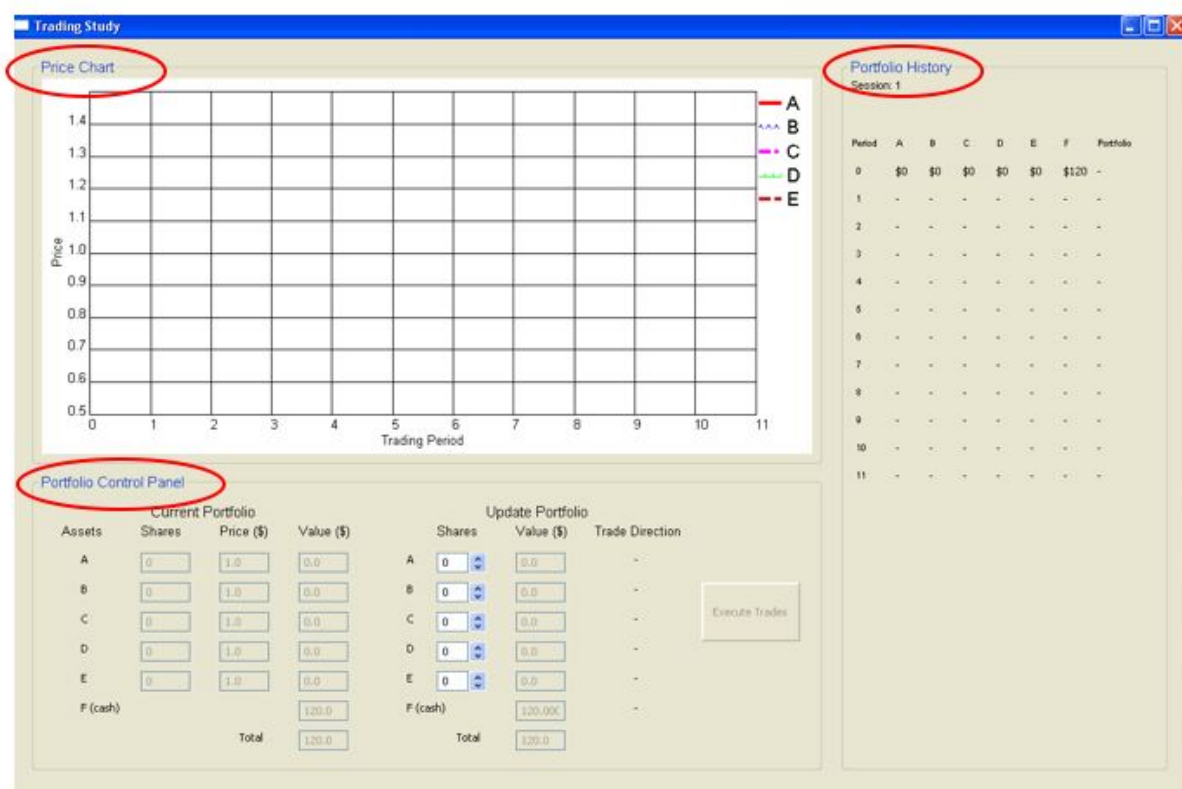
Example 6: What is true about cash? Mark all that apply.

**Answer:**

- ☐ Cash can be held in all periods
- ☐ The value of cash will stay constant
- ☐ Cash is always called asset F
- ☐ All of the above

## Your trading screen

This is what your screen will look like. There are three main parts to the screen: The price chart, the portfolio control panel and the portfolio history.



**1. Price chart:** This chart will be empty when an investment session starts. As you buy and sell and move through the trading periods, the prices of assets A, B, C, D, and E will be visible in the chart.

**2. Portfolio Control Panel:** The left part of this panel is the current portfolio. There you can see what assets you have, how many of each, how much you have in cash, and what the price of each asset is. You make your buying and selling decisions in the right part of the panel, "update portfolio". Use the arrows to increase or decrease the number of assets.

Whatever money you don't have in any of the assets A, B, C, D or E will automatically be in cash, asset F. Note that when you click the up-arrow, and the number of shares is increasing you are buying. When you click the down-arrow, and the number of shares is decreasing, you are selling. When you click on 'Execute Trades' you will make the changes (if any) that you put into the "update portfolio" part of the panel. You will then also see how the prices change, and if the value of your assets went down or up, i.e. if you gained or lost money from your investments.

**3. Portfolio History:** For your convenience you can check the portfolio history table if you want to be reminded of what your holdings were in the different trading periods. You can see your history for the current investment session but not for past investment sessions.

### Paper Record<sup>32</sup>

You have received a form labeled "Paper Record". On this form you should, in each investment session, circle the three assets that you have in trading period 0. When you have reached trading period 1 you should note on the form if the value of these assets went up or down by circling either the plus (if the value of the asset increased) or the minus (if the value of the asset decreased). This is illustrated below for the case where assets B, C and F were held in trading period 0 and the value of B went up and the value of C went down between trading period 0 and 1.

Investment session 1

Holdings from trading period 0:	A	<b>B</b>	<b>C</b>	D	E	<b>F</b>
Change btw period 0 and 1:	+ / -	<b>+</b> / -	+ / <b>-</b>	+ / -	+ / -	0

---

<sup>32</sup>The instructions under this heading were only given to subjects in the "writing" treatment group.

## **Going forward**

We will now start with investment session 1. As we pointed out before, this session is not for payment but for practice only. When you have finished the first session, please do NOT continue until we tell you to do so. We will wait until everyone has finished investment session 1 and then answer any questions before we move on to investment sessions 2 to 12.

### 4.2.3 Paper Record

Investment session 1	My holdings from trading period 0:	A	B	C	D	E	F
		+ / -	+ / -	+ / -	+ / -	+ / -	0
Investment session 2	My holdings from trading period 0:	A	B	C	D	E	F
		+ / -	+ / -	+ / -	+ / -	+ / -	0
Investment session 3	My holdings from trading period 0:	A	B	C	D	E	F
		+ / -	+ / -	+ / -	+ / -	+ / -	0
Investment session 4	My holdings from trading period 0:	A	B	C	D	E	F
		+ / -	+ / -	+ / -	+ / -	+ / -	0
Investment session 5	My holdings from trading period 0:	A	B	C	D	E	F
		+ / -	+ / -	+ / -	+ / -	+ / -	0
Investment session 6	My holdings from trading period 0:	A	B	C	D	E	F
		+ / -	+ / -	+ / -	+ / -	+ / -	0
Investment session 7	My holdings from trading period 0:	A	B	C	D	E	F
		+ / -	+ / -	+ / -	+ / -	+ / -	0
Investment session 8	My holdings from trading period 0:	A	B	C	D	E	F
		+ / -	+ / -	+ / -	+ / -	+ / -	0
Investment session 9	My holdings from trading period 0:	A	B	C	D	E	F
		+ / -	+ / -	+ / -	+ / -	+ / -	0
Investment session 10	My holdings from trading period 0:	A	B	C	D	E	F
		+ / -	+ / -	+ / -	+ / -	+ / -	0
Investment session 11	My holdings from trading period 0:	A	B	C	D	E	F
		+ / -	+ / -	+ / -	+ / -	+ / -	0
Investment session 12	My holdings from trading period 0:	A	B	C	D	E	F
		+ / -	+ / -	+ / -	+ / -	+ / -	0



#### 4.2.4 Post-Experimental Questionnaire

How did you decide which shares to buy and sell? Did you use any “rule of thumb” for deciding when to trade and how to trade?

---

---

---

When you were deciding whether or not it would be a good idea to sell a share, which price did you compare the current price of the share to?

- ☐ The price that I bought the share for.
- ☐ The price in the previous trading period.
- ☐ The price that I think the share is worth.
- ☐ Another price. Please explain which: \_\_\_\_\_
- ☐ I didn't compare the current price to any price in particular.

What is your experience from acting on financial markets and/or investing? Please check all that apply.

- ☐ Work in financial industry
- ☐ Studied financial economics in college / graduate school
- ☐ Trading with own money
- ☐ Other: \_\_\_\_\_

Please list the names of the courses that you have taken in economics, finance, econometrics and statistics:

---

---

---

Gender: ☐ Female ☐ Male

Born in year: \_\_\_\_\_

## 4.3 Chapter 3 Appendix

### 4.3.1 Optimal Demands and Market-Clearing Prices

Investors have quadratic utility over next-period wealth:

$$U(W_{t+1}) = \mathbb{E}_t[W_{t+1}] - \frac{\gamma}{2} \text{Var}_t[W_{t+1}]$$

Investors myopically optimize their portfolios with respect to this utility function in each period. At time  $t$ , investors believe  $V \sim N(\mathbb{E}_t^M[V], \text{Var}_t[V])$ , where  $\mathbb{E}_t^M[V]$  and  $\text{Var}_t[V]$  are the expectation and variance, respectively, of fundamental value, conditional upon public information available at time  $t$ .

At each date  $t$ , investors solve for the optimal amount to invest in the stock,  $x_t^*$ :

$$\begin{aligned} & \max_x \left\{ \mathbb{E}_t[W_{t+1}] - \frac{\gamma}{2} \text{Var}_t[W_{t+1}] \right\} \\ = & \max_x \left[ (\mathbb{E}_t^M[V - P_t] x) - \frac{\gamma}{2} \text{Var}_t[V - P_t] x^2 \right] \\ = & \max_x \left[ (\mathbb{E}_t^M[V] - P_t) x - \frac{\gamma}{2} \text{Var}_t[V] x^2 \right] \\ \iff & \max_x \left\{ (\mathbb{E}_t^M[V] - P_t) x - \frac{\gamma}{2} \text{Var}_t[V] x^2 \right\} \\ \iff & (\mathbb{E}_t^M[V] - P_t) - \gamma \text{Var}_t[V] x = 0 \\ \iff & \mathbb{E}_t^M[V] - P_t = \gamma \text{Var}_t[V] x \\ & x_t^* = \frac{\mathbb{E}_t^M[V] - P_t}{\gamma \text{Var}_t[V]} \end{aligned}$$

Market-clearing then requires

$$\begin{aligned}
P_t &= Nx_t^* \\
P_t &= \frac{N(\mathbb{E}_t^M[V] - P_t)}{\gamma \text{Var}_t[V]} \\
\frac{\gamma \text{Var}_t[V]}{N} P_t &= \mathbb{E}_t^M[V] - P_t \\
P_t \left(1 + \frac{\gamma \text{Var}_t[V]}{N}\right) &= \mathbb{E}_t^M[V] \\
P_t &= \frac{N \mathbb{E}_t^M[V]}{N + \gamma \text{Var}_t[V]} \\
\Rightarrow x_t^* &= \frac{\mathbb{E}_t^M[V]}{N + \gamma \text{Var}_t[V]}
\end{aligned}$$

#### 4.3.1.1 PIT Portfolio Choice and Trading

Consider the portfolio optimization problem facing PIT. At date  $t$ , he solves

$$\begin{aligned}
&\max_x \left\{ \tau_t \mathbb{E}[(V - P_t)x | V = S] + (1 - \tau_t) \left( (\mathbb{E}_t^M[V - P_t]x) - \frac{\gamma}{2} \text{Var}_t[V - P_t]x^2 \right) \right\} \\
&= \max_x \left\{ \tau_t ((S - P_t)x) + (1 - \tau_t) \left( (\mathbb{E}_t^M[V] - P_t)x - \frac{\gamma}{2} \text{Var}_t[V]x^2 \right) \right\} \\
&= \max_x \left\{ \tau_t (S - P_t)x + (\mathbb{E}_t^M[V] - P_t)x - \tau_t (\mathbb{E}_t^M[V] - P_t)x - \frac{\gamma}{2} \text{Var}_t[V]x^2 + \tau_t \frac{\gamma}{2} \text{Var}_t[V]x^2 \right\} \\
\text{FOC : } &\tau_t (S - P_t) + (\mathbb{E}_t^M[V] - P_t) - \tau_t (\mathbb{E}_t^M[V] - P_t) - (1 - \tau_t)\gamma \text{Var}_t[V]x = 0 \\
&\tau_t (S - P_t) + (1 - \tau_t) (\mathbb{E}_t^M[V] - P_t) = (1 - \tau_t)\gamma \text{Var}_t[V]x \\
x_t^\dagger &= \frac{\tau_t (S - P_t) + (1 - \tau_t) (\mathbb{E}_t^M[V] - P_t)}{(1 - \tau_t)\gamma \text{Var}_t[V]} \\
x_t^\dagger &= \frac{\tau_t}{(1 - \tau_t)} \frac{S - P_t}{\gamma \text{Var}_t[V]} + \frac{\mathbb{E}_t^M[V] - P_t}{\gamma \text{Var}_t[V]}
\end{aligned}$$

We can express PIT's optimal allocation to the stock,  $x_t^\dagger$  in terms of the average investor's optimal allocation,  $x_t^*$ :

$$x_t^\dagger = \frac{\tau_t}{(1 - \tau_t)} \frac{S - P_t}{\gamma \text{Var}_t[V]} + x_t^*$$

It will also be useful to have an explicit expression for PIT's excess demand, relative to that of an average investor, so define

$$\begin{aligned}
\Delta_t &\equiv x_t^\dagger - x_t^* \\
&= \frac{\tau_t (S - \mathbb{E}_t^M[V])}{(1 - \tau_t)\gamma \text{Var}_t[V]} + \frac{1}{1 - \tau_t} x_t^* - x_t^* \\
&= \frac{\tau_t (S - \mathbb{E}_t^M[V])}{(1 - \tau_t)\gamma \text{Var}_t[V]} + \left( \frac{1}{1 - \tau_t} - \frac{1 - \tau_t}{1 - \tau_t} \right) x_t^* \\
\Delta_t &= \frac{\tau_t (S - \mathbb{E}_t^M[V])}{(1 - \tau_t)\gamma \text{Var}_t[V]} + \frac{\tau_t}{1 - \tau_t} x_t^* \\
&= \frac{\tau_t}{(1 - \tau_t)} \frac{S - P_t}{\gamma \text{Var}_t[V]}
\end{aligned}$$

### 4.3.2 Investors' Belief Updating

Let  $\psi$  denote the realized value of  $\Psi$ ; following this revelation, non-PIT investors update their priors and assign  $V$  a posterior (Gaussian) distribution with the following moments:

$$\begin{aligned}
\mathbb{E}^M[V|\Psi = \psi] &= \mu + \frac{\sigma_V^2}{\sigma_V^2 + \sigma_\Psi^2} (\psi - \mu) \\
&= \mu + \frac{\sigma_V^2}{\sigma_V^2 + \sigma_\epsilon^2} (\psi - \mu) \\
&= \frac{\sigma_\epsilon^2 \mu + \sigma_V^2 \psi}{\sigma_V^2 + \sigma_\epsilon^2} \\
\text{Var}^M[V|\Psi = \psi] &= \frac{1}{\frac{1}{\sigma_V^2} + \frac{1}{\sigma_\epsilon^2}} \\
&= \frac{\sigma_\epsilon^2 \sigma_V^2}{\sigma_\epsilon^2 + \sigma_V^2} \\
&= \left( \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_V^2} \right) \sigma_V^2
\end{aligned}$$

The updating problem facing PIT is clearly different, since his prior beliefs about  $V$  differ from those of the other investors. PIT uses the signal  $\psi$  to update his estimate of the probability  $\tau$  that his signal is true. If PIT's signal is true, then  $\Psi \sim N(S, \sigma_\epsilon^2)$ , while if his signal is false,  $\Psi \sim N(\mu, \sigma_V^2 + \sigma_\epsilon^2)$ , so we have the following conditional densities for  $\Psi = \psi$ :

$$\begin{aligned}
g(\psi|true) &= \frac{1}{\sqrt{2\pi\sigma_\epsilon^2}} \exp\left(-\frac{(\psi - S)^2}{2\sigma_\epsilon^2}\right) \\
g(\psi|false) &= \frac{1}{\sqrt{2\pi(\sigma_\epsilon^2 + \sigma_V^2)}} \exp\left(-\frac{(\psi - \mu)^2}{2(\sigma_\epsilon^2 + \sigma_V^2)}\right)
\end{aligned}$$

Now, define the likelihood ratio

$$\begin{aligned}
\Lambda_\psi &\equiv \frac{g(\psi|true)}{g(\psi|false)} \\
&= \frac{\sqrt{2\pi(\sigma_\epsilon^2 + \sigma_V^2)} \exp\left(-\frac{(\psi - S)^2}{2\sigma_\epsilon^2}\right)}{\sqrt{2\pi\sigma_\epsilon^2} \exp\left(-\frac{(\psi - \mu)^2}{2(\sigma_\epsilon^2 + \sigma_V^2)}\right)} \\
&= \sqrt{\frac{\sigma_\epsilon^2 + \sigma_V^2}{\sigma_\epsilon^2}} \exp\left(-\frac{(\psi - S)^2}{2\sigma_\epsilon^2} + \frac{(\psi - \mu)^2}{2(\sigma_\epsilon^2 + \sigma_V^2)}\right) \\
&= \sqrt{1 + \frac{\sigma_V^2}{\sigma_\epsilon^2}} \exp\left(\frac{\sigma_\epsilon^2 (\psi - \mu)^2 - (\sigma_\epsilon^2 + \sigma_V^2) (\psi - F)^2}{2\sigma_\epsilon^2 (\sigma_\epsilon^2 + \sigma_V^2)}\right)
\end{aligned}$$

Using Bayes' rule, PIT updates his estimate of  $\tau$ :

$$\tau_1 = \frac{\Lambda_\psi \tau_0}{\Lambda_\psi \tau_0 + (1 - \tau_0)}$$

Note that  $\tau_1 > \tau_0 \iff \Lambda_\psi > 1$ .

#### 4.3.2.1 Investor Trading Following the Revelation of $\psi$

After observing  $\Psi$  and updating their beliefs, each non-PIT investor has a demand for the stock given by

$$\begin{aligned}
x_1^* &= \frac{\mathbb{E}_1^M[V] - P_1}{\gamma \text{Var}_1[V]} \\
&= \frac{\mu + \frac{\sigma_V^2}{\sigma_V^2 + \sigma_\epsilon^2}(\psi - \mu) - P_1}{\gamma \left( \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_V^2} \right) \sigma_V^2} \\
&= \frac{(\sigma_\epsilon^2 + \sigma_V^2)(\mu - P_1) + \sigma_V^2(\psi - \mu)}{\gamma \sigma_\epsilon^2 \sigma_V^2} \\
&= \frac{(\sigma_\epsilon^2 \mu + \sigma_V^2 \psi) - (\sigma_\epsilon^2 + \sigma_V^2) P_1}{\gamma \sigma_\epsilon^2 \sigma_V^2} \\
&= \frac{\sigma_\epsilon^2 \mu + \sigma_V^2 \psi}{\gamma \sigma_\epsilon^2 \sigma_V^2} - \frac{\left( \frac{\sigma_\epsilon^2 + \sigma_V^2}{\sigma_\epsilon^2 \sigma_V^2} \right) P_1}{\gamma} \\
&= \frac{\sigma_\epsilon^2 \mu + \sigma_V^2 \psi}{\gamma \sigma_\epsilon^2 \sigma_V^2} - \frac{\pi_\epsilon + \pi_V}{\gamma} P_1
\end{aligned}$$

where  $\pi_\epsilon \equiv 1/\sigma_\epsilon^2$  and  $\pi_V \equiv 1/\sigma_V^2$  are precisions.

Then by market-clearing (assuming that the excess demand by PIT is negligible, and ignoring noise traders), we have

$$\begin{aligned}
P_1 &= Nx_1^* \\
P_1 &= N \frac{(\sigma_\epsilon^2 \mu + \sigma_V^2 \psi) - (\sigma_\epsilon^2 + \sigma_V^2) P_1}{\gamma \sigma_\epsilon^2 \sigma_V^2} \\
\frac{\gamma \sigma_\epsilon^2 \sigma_V^2}{N} P_1 &= (\sigma_\epsilon^2 \mu + \sigma_V^2 \psi) - (\sigma_\epsilon^2 + \sigma_V^2) P_1 \\
\frac{\gamma \sigma_\epsilon^2 \sigma_V^2}{N (\sigma_\epsilon^2 + \sigma_V^2)} P_1 &= \mathbb{E}_1^M [V] - P_1 \\
P_1 \left( 1 + \frac{\gamma \sigma_\epsilon^2 \sigma_V^2}{N (\sigma_\epsilon^2 + \sigma_V^2)} \right) &= \mathbb{E}_1^M [V] \\
P_1 \left( \frac{N (\sigma_\epsilon^2 + \sigma_V^2) + \gamma \sigma_\epsilon^2 \sigma_V^2}{N (\sigma_\epsilon^2 + \sigma_V^2)} \right) &= \mathbb{E}_t^M [V] \\
P_1 &= \frac{N (\sigma_\epsilon^2 + \sigma_V^2)}{N (\sigma_\epsilon^2 + \sigma_V^2) + \gamma \sigma_\epsilon^2 \sigma_V^2} \mathbb{E}_1^M [V] \\
&= \frac{N (\sigma_\epsilon^2 \mu + \sigma_V^2 \psi)}{N (\sigma_\epsilon^2 + \sigma_V^2) + \gamma \sigma_\epsilon^2 \sigma_V^2} \\
\Rightarrow x_1^* &= \frac{(\sigma_\epsilon^2 + \sigma_V^2)}{N (\sigma_\epsilon^2 + \sigma_V^2) + \gamma \sigma_\epsilon^2 \sigma_V^2} \mathbb{E}_1^M [V] \\
&= \frac{\sigma_\epsilon^2 \mu + \sigma_V^2 \psi}{N (\sigma_\epsilon^2 + \sigma_V^2) + \gamma \sigma_\epsilon^2 \sigma_V^2}
\end{aligned}$$

However, because the supply of the stock is fixed and the non-PIT investors are assumed to be identical, it follows from the symmetry of the situation (again assuming that the excess demand by PIT is negligible, and ignoring the effects noise traders) that

$$\frac{x_0^*}{P_0} = \frac{x_1^*}{P_1} = \frac{1}{N}$$

For PIT (whose demand, we assume, has a negligible impact on market price), new optimal

demand for the stock is given by

$$\begin{aligned}
x_1^\dagger &= \frac{\tau_1(S - P_1) + (1 - \tau_1) (\mathbb{E}_1^M[V] - P_1)}{(1 - \tau_1)\gamma \left( \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_V^2} \right) \sigma_V^2} \\
&= (\pi_\epsilon + \pi_V) \frac{\tau_1(S - P_1) + (1 - \tau_1) (\mathbb{E}_1^M[V] - P_1)}{(1 - \tau_1)\gamma} \\
&= (\pi_\epsilon + \pi_V) \frac{\tau_1(S - P_1)}{(1 - \tau_1)\gamma} + x_1^*
\end{aligned}$$

The number of shares of the stock that PIT demands at  $t = 1$  is given by

$$\begin{aligned}
\frac{x_1^\dagger}{P_1} &= (\pi_\epsilon + \pi_V) \frac{\tau_1(S - P_1)}{(1 - \tau_1)\gamma P_1} + \frac{x_1^*}{P_1} \\
&= \frac{1}{N} + \frac{\pi_\epsilon + \pi_V}{\gamma} \frac{\tau_1}{1 - \tau_1} \frac{S - P_1}{P_1}
\end{aligned}$$

Similarly, the number of shares of the stock that PIT demands at  $t = 0$  is given by

$$\begin{aligned}
\frac{x_0^\dagger}{P_0} &= \frac{\tau_0(S - P_0) + (1 - \tau_0) (\mathbb{E}_0^M[V] - P_0)}{P_0(1 - \tau_0)\gamma\sigma_V^2} \\
&= \frac{\tau_0(S - P_0)}{P_0(1 - \tau_0)\gamma\sigma_V^2} + \frac{x_0^*}{P_0} \\
&= \frac{1}{N} + \frac{\pi_V}{\gamma} \frac{\tau_0}{1 - \tau_0} \frac{S - P_0}{P_0}
\end{aligned}$$

Next, using the definition  $\Delta_t \equiv x_t^\dagger - x_t^*$ , and noting that

$$\frac{\Delta_t}{P_t} = \frac{x_t^\dagger - x_t^*}{P_t} = \frac{x_t^\dagger}{P_t} - \frac{1}{N}$$



we have

$$\begin{aligned}
\frac{\Delta_1/P_1}{\Delta_0/P_0} &= \frac{\frac{\pi_\epsilon + \pi_V}{\gamma} \frac{\tau_1}{1-\tau_1} \frac{S-P_1}{P_1}}{\frac{\pi_V}{\gamma} \frac{\tau_0}{1-\tau_0} \frac{S-P_0}{P_0}} \\
&= \frac{\pi_\epsilon + \pi_V}{\pi_V} \frac{\tau_1}{1-\tau_1} \frac{1-\tau_0}{\tau_0} \frac{P_0}{P_1} \left( \frac{S-P_1}{S-P_0} \right) \\
&= \frac{\pi_\epsilon + \pi_V}{\pi_V} \frac{\tau_1}{\tau_0} \frac{1-\tau_0}{1-\tau_1} \frac{P_0}{P_1} \left( \frac{S-P_1}{S-P_0} \right) \\
&= \frac{\pi_\epsilon + \pi_V}{\pi_V} \frac{P_0}{P_1} \left( \frac{S-P_1}{S-P_0} \right) \frac{\frac{\Lambda_\psi \tau_0}{\Lambda_\psi \tau_0 + (1-\tau_0)}}{\tau_0} \frac{1-\tau_0}{1 - \frac{\Lambda_\psi \tau_0}{\Lambda_\psi \tau_0 + (1-\tau_0)}} \\
&= \frac{\pi_\epsilon + \pi_V}{\pi_V} \frac{P_0}{P_1} \left( \frac{S-P_1}{S-P_0} \right) \frac{\Lambda_\psi \tau_0}{(\Lambda_\psi \tau_0 + (1-\tau_0)) \tau_0} \frac{(1-\tau_0)(\Lambda_\psi \tau_0 + (1-\tau_0))}{(1-\tau_0)} \\
&= \frac{\pi_\epsilon + \pi_V}{\pi_V} \frac{P_0}{P_1} \left( \frac{S-P_1}{S-P_0} \right) \Lambda_\psi \\
&= \frac{\pi_\epsilon + \pi_V}{\pi_V} \Lambda_\psi \frac{\frac{S-P_1}{P_1}}{\frac{S-P_0}{P_0}}
\end{aligned}$$

# Bibliography

- [1] Nicholas Barberis and Ming Huang. Mental accounting, loss aversion, and individual stock returns. *The Journal of Finance*, 56(4), August 2001.
- [2] Nicholas Barberis, Ming Huang, and Richard H. Thaler. Individual preferences, monetary gambles, and stock market participation: A case for narrow framing. *The American Economic Review*, 96(4):1069–1090, 2006.
- [3] Nicholas Barberis and Wei Xiong. What drives the disposition effect? an analysis of a long-standing preference-based explanation. *The Journal of Finance*, 64:751–784, April 2009.
- [4] Matthew Baron, Jonathan Brogaard, and Andrei Kirilenko. The trading profits of high frequency traders. September 2012.
- [5] Itzhak Ben-David and David Hirshleifer. Are investors really reluctant to realize their losses? trading responses to past returns and the disposition effect. *Review of Financial Studies*, 25(8):2485–2532, 2012.
- [6] Bruno Biais, Thierry Foucault, and Sophie Moinas. Equilibrium algorithmic trading. Working Paper, October 2010.
- [7] Jonathan Brogaard, Terrence Hendershott, and Ryan Riordan. High frequency trading and price discovery. 2012.
- [8] Laurent E. Calvet, John Y. Campbell, and Paolo Sodini. Fight or flight? portfolio rebalancing by individual investors. *Quarterly Journal of Economics*, 124(1):301–348, 2009.
- [9] Giovanni Cespa and Thierry Foucault. Insiders-outsiders, transparency and the value of the ticker. Queen Mary University Dept. of Economics Working Paper No. 628, April 2008.
- [10] Tom Chang, David H. Solomon, and Mark M. Westerfield. Looking for someone to blame: Delegation, cognitive dissonance and the disposition effect. December 2012.
- [11] Lauren Cohen, Andrea Frazzini, and Christopher Malloy. Sell side school ties. NBER Working Papers 13973, National Bureau of Economic Research, Inc, May 2008.
- [12] J. Bradford de Long, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann. Positive feedback investment strategies and destabilizing rational speculation. *The Journal of Finance*, 45(2):379–395, June 1990.

- [13] Alfonso Dufour and Robert F. Engle. Time and the price impact of a trade. *The Journal of Finance*, 55(6):2467–2498, December 2000.
- [14] David Easley and Nicholas M. Kiefer. Optimal learning with endogenous data. *International Economic Review*, 30(4):963–978, November 1989.
- [15] David Easley and Maureen O’Hara. Time and the process of security price adjustment. *The Journal of Finance*, 47(2):577–605, 1992.
- [16] Simon Gervais. Market microstructure with uncertain information precision: A multi-period analysis. January 1996.
- [17] Lawrence R. Glosten and Paul R. Milgrom. Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. *The Journal of Financial Economics*, 14:71–100, 1985.
- [18] Sanford Grossman. On the efficiency of competitive stock markets where trades have diverse information. *The Journal of Finance*, 31(2):573–585, May 1976.
- [19] Sanford J. Grossman and Merton H. Miller. Liquidity and market structure. *The Journal of Finance*, 43(3), July 1988.
- [20] Sanford J. Grossman and Joseph E. Stiglitz. On the impossibility of informationally efficient markets. *The American Economic Review*, 70(3):393–408, 1980.
- [21] J. Michael Harrison and David M. Kreps. Speculative investor behavior in a stock market with heterogeneous expectations. *The Quarterly Journal of Economics*, 92(2):323–336, 1978.
- [22] Joel Hasbrouck and Gideon Saar. Technology and liquidity provision: The blurring of traditional definitions. *Journal of Financial Markets*, 12:143–172, 2009.
- [23] Joel Hasbrouck and Gideon Saar. Low-latency trading. NYU Stern/ Cornell GSM Working Paper, May 2011.
- [24] Terrence Hendershott, Charles M. Jones, and Albert J. Menkveld. Does algorithmic trading improve liquidity? *The Journal of Finance*, 66(1), February 2011.
- [25] Terrence Hendershott and Ryan Riordan. Algorithmic trading and information. NET Institute Working Paper No. 09-08, September 2009.
- [26] Harrison Hong, Jeffrey D. Kubik, and Jeremy C. Stein. The only game in town: Stock-price consequences of local bias. NBER Working Papers 11488, National Bureau of Economic Research, Inc, July 2005.
- [27] Robert Jarrow and Philip Protter. A dysfunctional role of high frequency trading in electronic markets. Johnson School Research Paper Series No. 08-2011, March 2011.
- [28] Daniel Kahneman and Amos Tversky. Prospect theory: An analysis of decision under risk. *Econometrica*, 47(2):263–291, 1979.

- [29] Daniel Kahneman and Amos Tversky. Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, 5:297–323, 1992.
- [30] Andrei Kirilenko, Mehrdad Samadi, Albert S. Kyle, and Tugkan Tuzun. The flash crash: The impact of high frequency trading on an electronic market. October 2010.
- [31] Albert S. Kyle. Continuous auctions and insider trading. *Econometrica*, 53(6):1315–1335, November 1985.
- [32] J.Chris Leach and Ananth N Madhavan. Intertemporal price discovery by market makers: Active versus passive learning. *Journal of Financial Intermediation*, 2(2):Pages 207–235, June 1992.
- [33] J.Chris Leach and Ananth N. Madhavan. Price experimentation and security market structure. *The Review of Financial Studies*, 6(2):pp. 375–404, 1993.
- [34] Giuseppe Moscarini and Lones Smith. The optimal level of experimentation. *Econometrica*, 69(6):1629–1644, November 2001.
- [35] Terrence Odean. Are investors reluctant to realize their losses? *The Journal of Finance*, 53(5):1775–1798, October 1998.
- [36] Matthew Rabin. Risk aversion and expected-utility theory: A calibration theorem. *Econometrica*, 68(5):1281–1292, September 2000.
- [37] Elena Rangelova. Disposition effect and firm size: New evidence on individual investor trading activity. Harvard University, 2000.
- [38] Alvin E. Roth. *The handbook of experimental economics*, volume 1. Princeton university press, 1995.
- [39] U.S. SEC. Concept release on equity market structure, concept release no. 34-61358. FileNo. 17 CFR Part 242 [Release No. 34-61358; File No. S7-02-10] RIN 3235-AK47, January 2010.
- [40] U.S. SEC. Findings regarding the market events of may 6, 2010, September 2010.
- [41] Hersh Shefrin and Meir Statman. The disposition to sell winners too early and ride losers too long: Theory and evidence. *The Journal of Finance*, 40(3):777–790, 1985.
- [42] Hans R. Stoll. Inferring the components of the bid-ask spread: Theory and empirical tests. *The Journal of Finance*, 44(1):115–134, March 1989.
- [43] M. Weber and C.F. Camerer. The disposition effect in securities trading: an experimental analysis. *Journal of Economic Behavior and Organization*, 33:167–184, 1998.
- [44] M. Zeelenberg, J. Beattie, J. van der Pligt, and N.K. de Vries. Consequences of regret aversion: Effect of expected feedback on risky decision making. *Organization Behavior and Human decision Processes*, 65(2):148–158, 1996.
- [45] M. Zeelenberg, W. W. Van Dijk, and A. S. R. Manstead. Reconsidering the relation between regret and responsibility. *Organizational Behavior and Human Decision Processes*, 74(3):254–272, 1998.